# A Simulation Study of Global Warming and Optimal Climate Policy*

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## Preliminary version – comments welcome!

**Abstract**

This paper presents a quantitative study of climate policies within a dynamic equilibrium model featuring multiple regions. The optimal climate policy consists of a Pigovian tax on carbon emissions and a transfer policy which distributes tax revenue across countries. We propose a generally applicable computational algorithm to compute the equilibrium solution under alternative climate policies and conduct a numerical case study which compares the consequences of climate change for OECD and Non-OECD countries. Optimal taxation keeps the increase in global temperature below two degrees and both regions sustain positive growth in the long run. Under Laissez faire where emissions are not taxed, temperature exceeds the two-degree target within the next forty years and increases exponentially thereafter, leading to massive damages and output losses in the long run. This result, however, hinges crucially on the emergence of a backstop technology which replaces coal within the next fifty years. Otherwise, climate damages under Laissez faire are still significant but much smaller, stressing the crucial role of this assumption in existing climate studies. Finally, we compute optimal transfer payments under a Pareto-improving transfer policy. While homogeneous climate damages imply moderate transfer payments from OECD to Non-OECD countries, the direction of transfers reverses as climate damages are increasingly biased towards Non-OECD countries. The intuition is that countries severely affected by climate damages are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfer payments.

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# Introduction

Global warming constitutes the biggest *economic*, *environmental*, and *social* challenge of the twenty-first century. To guide the ongoing political debate about potential solutions, economic theory plays a crucial role to assess the economic consequences of alternative climate policies. For this purpose, traditional macroeconomic models must be supplemented by a climate model describing the interactions between production activity and climate variables. Within such models, different policy proposals can be analyzed and compared. A simulation study of this type which analyzes the *economic*, *environmental*, and *social* consequences of alternative climate policies is presented in this paper.

Economic models which incorporate the interactions between climate variables and the economic production process are referred to as *integrated assessment (IA)* models in the literature. Most of the existing IA-models are based on the RICE/DICE framework pioneered by Nordhaus (1977) and further developed in Nordhaus & Yang (1996). The numerous extensions and refinements of the RICE/DICE model are comprehensively surveyed in Nordhaus (2011) and, more recently, in Hassler et al. (2016). A typical feature of this class of models which is stressed in Hassler et al. (2016) is that their solutions are essentially derived as planning problems making only limited use of the dynamic general equilibrium approach used in modern macroeconomics. Among other things, this makes it difficult to study possibly non-optimal tax policies and, therefore, to evaluate the economic consequences of arbitrary climate policies.

A framework which takes full advantage of dynamic general equilibrium theory is developed in Golosov et al. (2014), henceforth GHKT. Their main theoretical result is an explicit formula for the optimal tax on fossil emissions which provides an important benchmark for current simulation studies. In addition, they conduct a comprehensive simulation study of alternative climate policies. A major limitation of the GHKT model is that it treats the world as a single region and, therefore, does not allow to study the impact of global warming and alternative climate policies on different regions. While an extension of the GHKT model to a multi-region framework is developed in Hassler & Krusell (2012), they impose strong restrictions on trade between regions which are only allowed to trade fossil energy inputs to preserve analytical tractability.

An alternative model with multiple regions is developed in Hillebrand & Hillebrand (2017), henceforth HH. Adopting the same modelling philosophy as GHKT, this model distinguishes an arbitrary number of regions which differ with respect to their state of economic development, factor endowments, and climate damages. At the same time, the HH-model avoids the restrictions on trade in Hassler & Krusell (2012) and allows for countries to interact on global markets for goods, capital, and exhaustible resources. This is accomplished by dividing the production process in each region into three stages: The final production stage produces the consumption good, the energy stage which provides energy goods and services to final production, and the resource stage supplies exhaustible resources to energy production. At the same time, the model retains the virtue of analytical tractability and permits to determine an optimal climate policy consisting of an optimal emissions tax and a transfer policy. The latter determines how tax revenues are redistributed as lump sum transfers to consumers in each region. While the optimal tax policy is unique and can even be characterized explicitly, the transfer policy depends on the weights placed on the interests of different countries and is therefore the crucial object to be determined in the political process.

The present paper uses the HH-model to conduct a numerical case study of alternative climate policies. Specifically, we compare the optimal policy to the Laissez faire solution where no taxation takes place. The analysis consists of two parts. The first part identifies the forward-recursive structure of the model and develops a generally applicable computational algorithm to compute the equilibrium solution under alternative climate policies. This sets the stage for our simulation study in the second part. Using calibrated parameter values, we divide the world in two regions (OECD and NOECD countries). For each scenario and region, we compare the *economic* consequences of climate policies such as their impact on output, production, and growth and their *environmental* consequences for climate variables such as CO2 emissions and global temperature.

The *social* aspect of climate policy in our study is represented by the transfer payments between regions. To study optimal transfers between regions, our study confines attention to a particular policy which determines transfers such that each region attains the same share of world consumption as under Laissez faire. This policy was shown in HH to lead to a Pareto improvement relative to the Laissez faire solution. While our benchmark scenario assumes homogeneous climate damages, we also compute optimal transfers under alternative scenarios where climate damages are increasingly biased towards NOECD countries. The idea that climate damages are asymmetric and biased towards developed countries has wide empirical support, cf. World Bank (2010).

In our simulations, we find that homogeneous climate damages imply moderate transfer payments from OECD to Non-OECD countries. However, the direction of transfers reverses as climate damages are increasingly biased towards Non-OECD countries. Thus, regions which suffer *more* damage from climate change receive *less* transfers. The intuition for this surprising (and potentially controversial) result is that countries severely affected by climate damages are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfer payments.

A key assumption made in our benchmark scenario and most quantitative studies of climate change is the existence of a backstop technology for coal. Originally introduced by Nordhaus (1973), such a technology assumes that coal use is not constrained by exhaustibility and, therefore, has no scarcity rent rendering its supply effectively infinite. Although widely used and analyzed in the literature (cf. Tahvonen & Salo (2001), Tsur & Zemel (2005), Chakravorty, Leach & Moreaux (2012), Valente (2011), and GHKT), the assumption remains somewhat controversial. Thus, a natural experiment is to analyze how the numerical results change in the absence of a backstop technology. This constitutes the final part of our simulation study.

While retaining the qualitative results from the benchmark scenario, we find that the quantitative results change substantially and climate damages are much more moderate under Laissez faire in the absence of a backstop technology. As the optimal solution in our benchmark scenario does not exhaust the existing stock of coal, it is not affected by this modification. We infer that the quantitative results on global warming under Laissez faire from the benchmark scenario hinge crucially on the hypothesized emergence of a backstop technology. Ironically, a more optimistic view about the abundance of exhaustible resources leads to a much more pessimistic prospect for the climate problem.

The paper is organized as follows. Section 1 introduces the model. Section 2 develops an algorithm to solve for the equilibrium under alternative specifications of a climate tax policy. Section 3 describes our calibration strategy of the model's parameters. The numerical results are presented in Sections 4 and 5 where we distinguish two scenarios depending on whether coal will be replaced by a backstop technology or not. Section 6 concludes, computational details are relegated to Appendix A.

# 1 The Model

Time evolves in discrete periods $t \in \mathbb{T} := \{0, 1, 2, \ldots\}$. The world economy is divided into $L \geq 1$ regions indexed by $\ell \in \mathbb{L} := \{1, \ldots, L\}$. Each region $\ell \in \mathbb{L}$ pursues its own interests and takes autonomous political decisions. Regions are geographically or institutionally separated, which imposes certain restrictions on trade between them.

The production process in each region $\ell \in \mathbb{L}$ decomposes into three stages. The first stage is the *final sector* which produces a consumable output commodity using labor, capital, and energy goods and services. The second stage consists of *energy sectors* which produce these goods and services using labor, capital, and exhaustible resources. The third stage consists of *resource sectors* which extract the domestic stock of exhaustible resources. The production side is complemented by a climate model and a description of the consumption sector in each region.

The following sections introduce these building blocks formally and derive the decentralized equilibrium solution for a given climate policy. See Hillebrand & Hillebrand (2017) for additional details on the model and the results presented in this section.

## 1.1 Production sectors

*Sectoral structure*
Production sectors in region $\ell \in \mathbb{L}$ are identified by the index $i \in \mathbb{I}_0 := \{0, 1, \ldots, I\}$.

Sector $i = 0$ is the *final sector* which produces a consumable output good in each period that can also be invested to become capital in the following period. Production sectors $i \in \mathbb{I} := \mathbb{I}_0 \backslash \{0\}$ are *energy sectors* which supply energy goods like electricity and heat or services like fuel-based transportation as inputs to final good production. We further denote by $\mathbb{I}_x \subset \mathbb{I}$ the set of energy sectors which base their production on exhaustible resource like coal, oil, and natural gas. As burning exhaustible resources in energy production causes emissions, sectors $\mathbb{I}_x$ are the sectors responsible for climate change. Production in the other energy sectors is based on renewable sources like wind, water, and solar energy which do not enter as production inputs and do not cause emissions.

Each sector $i \in \mathbb{I}_0$ consist of a single representative firm which employs labor $N_{i,t}^{\ell} \geq 0$ and capital $K_{i,t}^{\ell} \geq 0$ as production factors in period $t$. The amount of exhaustible resources used by sector $i \in \mathbb{I}_x$ is denoted $X_{i,t}^{\ell} \geq 0$ and is an essential input to production. All production technologies are based on time-invariant production functions $F_i$ which are linear homogeneous, twice continuously differentiable, strictly increasing, and concave.

Productivity in sector $i \in \mathbb{I}_0$ in region $\ell \in \mathbb{L}$ is denoted $Q_{i,t}^{\ell} > 0$ and may be time- and country-specific. Denote by $w_t^{\ell} > 0$ the wage and $p_{i,t}^{\ell} > 0$ the price of energy type $i \in \mathbb{I}$ in period $t$. As labor and energy outputs will be immobile across countries, their prices will, general, be region-specific. By contrast, capital and exhaustible resources are traded on international markets implying that their prices are not region-specific. The (rental) price of capital at time $t \geq 0$ is denoted $r_t > 0$ and the world price of the exhaustible resource used by sector $i \in \mathbb{I}_x$ as $v_{i,t} > 0$. Conceptually, all transactions take place in $t = 0$ and all prices in period $t$ are denominated in units of time $t$ consumption.

*Final sector*
Sector $i = 0$ in region $\ell \in \mathbb{L}$ uses labor, capital, and energy goods and services $(E_{i,t}^{\ell})_{i \in \mathbb{I}}$ to produce output $Y_t^{\ell}$ in period $t \geq 0$ according to the production technology

$$Y_t^{\ell} = (1 - D_t^{\ell}) Q_{0,t}^{\ell} F_0(K_{0,t}^{\ell}, N_{0,t}^{\ell}, (E_{i,t}^{\ell})_{i \in \mathbb{I}}). \tag{1}$$

Here, $D_t^{\ell} \in [0, 1[$ is an index of climate damage which will be a function of total $CO_2$-concentration in the atmosphere specified below. Given these parameters and prices for labor, capital, and energy inputs, the final sector solves the following atemporal decision problem in each period $t \geq 0$:

$$\max_{(K,N,E_1,\ldots,E_I) \in \mathbb{R}_+^{2+I}} \left\{ (1 - D_t^{\ell}) Q_{0,t}^{\ell} F_0(K, N, (E_i)_{i \in \mathbb{I}}) - w_t^{\ell} N - r_t K - \sum_{i \in \mathbb{I}} p_{i,t}^{\ell} E_i \right\}.$$

The profit maximizing solution in period $t \geq 0$ is characterized by the standard first order conditions:

$$\begin{aligned}
(1 - D_t^{\ell}) Q_{0,t}^{\ell} \partial_K F_0(K_{0,t}^{\ell}, N_{0,t}^{\ell}, (E_{i,t}^{\ell})_{i \in \mathbb{I}}) &= r_t && \text{(2a)} \\
(1 - D_t^{\ell}) Q_{0,t}^{\ell} \partial_N F_0(K_{0,t}^{\ell}, N_{0,t}^{\ell}, (E_{i,t}^{\ell})_{i \in \mathbb{I}}) &= w_t^{\ell} && \text{(2b)} \\
(1 - D_t^{\ell}) Q_{0,t}^{\ell} \partial_{E_i} F_0(K_{0,t}^{\ell}, N_{0,t}^{\ell}, (E_{i,t}^{\ell})_{i \in \mathbb{I}}) &= p_{i,t}^{\ell} \quad \forall i \in \mathbb{I}. && \text{(2c)}
\end{aligned}$$

*Exhaustible energy sectors*
Each sector $i \in \mathbb{I}_x$ is uniquely identified by the underlying resource on which production is based (like 'coal' used for 'coal-fired power generation' or 'oil' used to provide 'fuel-based transportation services'). The technology used by sector $i \in \mathbb{I}_x$ takes the form

$$E^{\ell}_{i,t} = Q^{\ell}_{i,t} F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}). \tag{3}$$

Burning exhaustible resources generates emissions proportional to their usage in production. Energy sectors thus represent the production stage at which emissions are potentially generated. The emissions generated from using $X^{\ell}_{i,t} \geq 0$ in production are $Z^{\ell}_{i,t} = \zeta_i X^{\ell}_{i,t}$ where $\zeta_i$ is the specific carbon-content of resource $i$. To combat climate damages, all regions impose a uniform climate tax $\tau_t \geq 0$ to be paid by energy sectors per unit of $CO_2$ emitted in period $t$. Firms in these sectors take this tax together with productivity and prices relevant to their decision as given parameters. Their decision problem solved in period $t \geq 0$ reads:

$$\max_{(K,N,X)\in\mathbb{R}^3_+} \left\{ p^{\ell}_{i,t} Q^{\ell}_{i,t} F_i(K, N, X) - w^{\ell}_t N - r_t K - (v_{i,t} + \tau_t \zeta_i) X \right\}.$$

Clearly, the profit maximizing solution becomes independent of $\tau_t$ if $\zeta_i = 0$, i.e., the firm employs a clean technology. The first order conditions necessary and sufficient for an optimal solution are given by:

$$p^{\ell}_{i,t} Q^{\ell}_{i,t} \partial_K F_i\big(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}\big) = r_t \tag{4a}$$
$$p^{\ell}_{i,t} Q^{\ell}_{i,t} \partial_N F_i\big(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}\big) = w^{\ell}_t \tag{4b}$$
$$p^{\ell}_{i,t} Q^{\ell}_{i,t} \partial_X F_i\big(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}\big) = v_{i,t} + \zeta_i \tau_t. \tag{4c}$$

*Renewable energy sectors*
Production of firms $i \in \mathbb{I} \backslash \mathbb{I}_x$ are based on *renewable sources* like wind, solar energy, etc. which do not enter as inputs to production. Their technology is given by

$$E^{\ell}_{i,t} = Q^{\ell}_{i,t} F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}) \tag{5}$$

Each firm $i \in \mathbb{I} \backslash \mathbb{I}_x$ in the renewable energy sector takes productivity and the relevant prices as given to solve the following profit maximization problem in period $t \geq 0$:

$$\max_{(K,N)\in\mathbb{R}^2_+} \left\{ p^{\ell}_{i,t} Q^{\ell}_{i,t} F_i(K, N) - w^{\ell}_t N - r_t K \right\}.$$

The first order conditions for profit maximization in period $t \geq 0$ are given by

$$p^{\ell}_{i,t} Q^{\ell}_{i,t} \partial_K F_i\big(K^{\ell}_{i,t}, N^{\ell}_{i,t}\big) = r_t \tag{6a}$$
$$p^{\ell}_{i,t} Q^{\ell}_{i,t} \partial_N F_i\big(K^{\ell}_{i,t}, N^{\ell}_{i,t}\big) = w^{\ell}_t. \tag{6b}$$

*Resource sectors*
Exhaustible resources are uniquely identified by the energy sector $i \in \mathbb{I}_x$ which uses this

resource in production. In each region $\ell \in \mathbb{L}$, there exists a single firm which extracts resources of type $i \in \mathbb{I}_x$. In period $t \geq 0$, this firm extracts resources $X_{i,t}^{\ell,s} \geq 0$ (to be distinguished from the amount $X_{i,t}^{\ell}$ demanded by energy sector $i \in \mathbb{I}_x$ in that region) and sells them in the global resource market at the price $v_{i,t}$. Firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R_{i,0}^{\ell} \geq 0$ together with the selling prices $(v_{i,t})_{t\geq 0}$ as a given parameter. Their objective is to maximize the discounted stream of future profits. As the economy is deterministic, profits in period $t \geq 0$ are discounted to period zero by the discount factor $q_t := \prod_{s=1}^{t} r_s^{-1}$ where $q_0 = 1$. With this notation, the decision problem solved by resource sector $i \in \mathbb{I}_x$ reads

$$\max_{(X_{i,t}^{\ell,s})_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} q_t(v_{i,t} - c_i)X_{i,t}^{\ell,s} \;\middle|\; \sum_{t=0}^{\infty} X_{i,t}^{\ell,s} \leq R_{i,0}^{\ell},\ X_{i,t}^{\ell,s} \geq 0\ \forall t \geq 0 \right\}.$$

If $R_{i,0}^{\ell} > 0$, the linearity of the extraction technology implies that an interior optimal extraction plan exists if and only if resource prices satisfy $v_{i,0} \geq 0$ and the Hotelling rule

$$v_{i,t} = c_i + r_t(v_{i,t-1} - c_i) \quad \forall t > 0. \tag{7}$$

Clearly, only if $v_{i,0} = c_i$ may it be optimal not to exhaust the entire stock of resources.In either case, (7) permits equilibrium profits of resource sector $i \in \mathbb{I}_x$ to be written as

$$\Pi_i^{\ell} = (v_{i,0} - c_i)R_{i,0}^{\ell}. \tag{8}$$

*Climate policy*
A climate policy determines the sequence of emissions taxes $\tau = (\tau_t)_{t\geq 0}$ which all regions are assumed to impose. The revenue from taxing emissions is then distributed as lump-sum transfers to consumers in each region. We assume that regions agree on a time-invariant transfer policy $\theta = (\theta^{\ell})_{\ell\in\mathbb{L}}$ satisfying $\sum_{\ell\in\mathbb{L}} \theta^{\ell} = 1$ which determines the share $\theta^{\ell}$ of tax revenue received by region $\ell$. This transfer policy constitutes the second part of a climate policy. Note that the case $\theta^{\ell} < 0$ is not excluded in this definition, in which case consumers in region $\ell$ are taxed to finance transfers received by other countries. Thus, the previous specification also allows for international redistribution via lump-sum taxation. It follows that the total discounted transfers received by consumers in region $\ell$ can be expressed as

$$T^{\ell} = \theta^{\ell} \sum_{t=0}^{\infty} q_t \tau_t \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^{\ell}. \tag{9}$$

## 1.2 Climate model

Emissions of $CO_2$ are generated by using ('burning') exhaustible fossil fuels like coal, oil, and gas in the production of energy. The amount of $CO_2$ generated by using one unit of exhaustible resource $i \in \mathbb{I}_x$ is physically determined by its carbon-content $\zeta_i \geq 0$. In particular, $\zeta_i = 0$ if the resource does no generate emissions, like uranium in the case of nuclear energy production. Total emissions in period $t$ measured in units of $CO_2$ are given by

$$Z_t := \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^{\ell}. \tag{10}$$

Adopting the specification from GHKT, the climate state in period $t$ consists of permanent and non-permanent $CO_2$ in the atmosphere and is denoted by $\mathbf{S}_t = (S_{1,t}, S_{2,t})$. Given an emissions sequence $\{Z_t\}_{t \geq 0}$ determined by (10), the climate state evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \tag{11a}$$
$$S_{2,t} = (1-\phi) S_{2,t-1} + (1-\phi_L)\phi_0 Z_t \tag{11b}$$

Specification (11) assumes that a share $0 \leq \phi_L < 1$ of emissions become permanent $CO_2$. Out of the remaining emissions, a share $\phi_0$ becomes non-permanent $CO_2$ which decays at constant rate $0 < \phi < 1$ while the remaining share $1 - \phi_0$ leaves the atmosphere (see GHKT for details). Total concentration of $CO_2$ at time $t$ is given by

$$S_t = S_{1,t} + S_{2,t}. \tag{12}$$

Denote by $\bar{S} > 0$ the pre-industrial level of $CO_2$ in the atmosphere. Climate damage in region $\ell$ is determined by total concentration of $CO_2$ in the atmosphere according to the function

$$D_t^{\ell} = D^{\ell}(S_t) := 1 - \exp\{-\gamma^{\ell}(S_t - \bar{S})\}, \quad \gamma^{\ell} > 0 \tag{13}$$

which corresponds to the choice in GHKT.[1] Reginal differences in climate damage thus enter via region specific parameters $\gamma^{\ell}$, $\ell \in \mathbb{L}$.

## 1.3 Consumption sector

The consumption sector in each region $\ell \in \mathbb{L}$ consists of a single representative household which supplies labor and capital to the production process and decides about consumption and capital formation taking factor prices as given. In addition, the consumer is entitled to receive all profits from domestic firms and transfers from the government. A direct consequence of the linear-homogeneity of the production functions $F_i$ is that profits in final production and all energy sectors are zero. Thus, by (8) the total discounted profit income of the household in region $\ell \in \mathbb{L}$ is

$$\Pi^{\ell} = \sum_{i \in \mathbb{I}_x} \Pi_i^{\ell} = \sum_{i \in \mathbb{I}_x} (v_{i,0} - c_i) R_{i,0}^{\ell}. \tag{14}$$

[1] The general version of GHKT allows for $\gamma$ to be time- and state-dependent. Here, we assume that it is constant, as they do in their numerical simulations, too.

The household's preferences over non-negative consumption sequences $(C_t^\ell)_{t\geq 0}$ are represented by a standard time-additive utility function

$$U((C_t^\ell)_{t\geq 0}) = \sum_{t=0}^{\infty} \beta^t u(C_t^\ell) \quad \text{ where } u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}, \quad \sigma > 0, 0 < \beta < 1. \tag{15}$$

Let $K_0^\ell$ denote the initial capital endowment in $t = 0$ and $\bar{N}_t^\ell > 0$ the labor supplied in period $t$ which is exogenous in our model. As before, let $q_t = \prod_{s=1}^{t} r_s^{-1}$ denote the discount factor for period $t$. Defining lifetime labor income $W^\ell := \sum_{t=0}^{\infty} q_t w_t^\ell \bar{N}_t^\ell$, transfer income $T^\ell$ as in (9), and profit income $\Pi^\ell$ as in (14), the consumer's decision problem reads:

$$\max_{(C_t^\ell)_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t^\ell) \;\middle|\; \sum_{t=0}^{\infty} q_t C_t^\ell \leq r_0 K_0^\ell + W^\ell + \Pi^\ell + T^\ell, \; C_t^\ell \geq 0 \; \forall t \geq 0 \right\}.$$

At equilibrium, consumption $C_t^\ell$ in region $\ell \in \mathbb{L}$ is given by a constant share of world consumption $\bar{C}_t := \sum_{\ell\in\mathbb{L}} C_t^\ell$ each period $t \geq 0$, i.e.,

$$C_t^\ell = \mu^\ell \bar{C}_t = \frac{r_0 K_0^\ell + W^\ell + \Pi^\ell + T^\ell}{\sum_{k\in\mathbb{L}} \left(r_0 K_0^k + W^k + \Pi^k + T^k\right)} \bar{C}_t. \tag{16}$$

The evolution of aggregate consumption is determined by the Euler equation

$$\bar{C}_{t+1} = (\beta r_{t+1})^{\frac{1}{\sigma}} \bar{C}_t \tag{17}$$

and must satisfy the transversality condition

$$\lim_{T\to\infty} \beta^T u'(\bar{C}_t) \bar{K}_{T+1} = 0 \tag{18}$$

where $\bar{K}_t$ is the aggregate world capital stock in period $t$.

## 1.4 Market clearing

As labor supply is immobile across regions, the labor market clearing condition for region $\ell$ in period $t$ reads

$$\sum_{i\in\mathbb{I}_0} N_{i,t}^\ell \overset{!}{=} \bar{N}_t^\ell. \tag{19}$$

By contrast, capital, exhaustible resources, and final output can freely be traded across countries. Letting $\bar{K}_t > 0$ denote the world capital stock in period $t$, market clearing on the global capital market requires

$$\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_0} K_{i,t}^\ell \overset{!}{=} \bar{K}_t \qquad \forall t \geq 0. \tag{20}$$

The market clearing condition for resource $i \in \mathbb{I}_x$ in period $t$ is $\sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell,s} \stackrel{!}{=} \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell}$. Summing over all countries, production inputs must satisfy the world exhaustible resource constraint

$$\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell} \leq R_{i,0} \qquad \forall i \in \mathbb{I}_x. \tag{21}$$

Here, $R_{i,0} := \sum_{\ell\in\mathbb{L}} R_{i,0}^{\ell}$ denotes the global initial stock of resource $i \in \mathbb{I}_x$. As the Hotelling rule (7) makes resource firms indifferent between the timing of extraction, the amount $X_{i,t}^{\ell,s}$ extracted in a particular region and period is, in general, indeterminate.

Finally, denoting world consumption by $\bar{C}_t$ as before, the world capital stock evolves as

$$\bar{K}_{t+1} = \sum_{\ell\in\mathbb{L}} Y_t^{\ell} - \bar{C}_t - \sum_{i\in\mathbb{I}_x} c_i \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell} \qquad \forall t \geq 0. \tag{22}$$

Equation (22) can be interpreted as a market clearing condition for final output.

## 1.5 Equilibrium

For $t \geq 0$, define the productivity vector $\mathbf{Q}_t := (Q_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$ and labor supply $\mathbf{N}_t^s := (\bar{N}_t^{\ell})_{\ell\in\mathbb{L}}$. The sequences $(\mathbf{Q}_t)_{t\geq 0}$ and $\mathbf{N}_t^s := (N_t^{\ell})_{\ell\in\mathbb{L}}$ are exogenously given in our model. Writing $\mathbf{Y}_t := (Y_t^{\ell})_{\ell\in\mathbb{L}}$, $\mathbf{E}_t := (E_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}}$, $\mathbf{K}_t := (K_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$, $\mathbf{N}_t := (N_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$, $\mathbf{X}_t := (X_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x}$, $\mathbf{S}_t := (S_{t,1}, S_{t,2})$, $\mathbf{w}_t := (w_t^{\ell})_{\ell\in\mathbb{L}}$, $\mathbf{p}_t := (p_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}}$, $\mathbf{v}_t := (v_{i,t})_{i\in\mathbb{I}_x}$, an aggregate equilibrium is a sequence $\xi = (\xi_t)_{t\geq 0}$ defined for each $t \geq 0$ as

$$\xi_t = (\mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, r_t, \mathbf{w}_t, \mathbf{p}_t, \mathbf{v}_t, \tau_t, \mathbf{S}_t, \bar{C}_t, \bar{K}_{t+1}) \tag{23}$$

which is consistent with the production technologies and optimality conditions (1)–(6) of producers, the Hotelling condition (7), the market clearing conditions (19), (20), (22) for labor, capital, and output, the global resource constraint (21), and climate conditions (10)–(13) as well as the Euler equation (17) and the transversality condition (18). The term aggregate is used because $\xi_t$ only involves aggregate consumption $\bar{C}_t$ but not its distribution across regions.

In the simulation study to be presented in this paper, two cases are of particular interest. First, the *Laissez faire equilibrium* $\xi^{\mathrm{LF}} = (\xi_t^{\mathrm{LF}})_{t\geq 0}$. This equilibrium represents the case where there is no attempt to correct market outcomes by imposing a climate tax. Due to the presence of a climate externality, this solution fails to be Pareto-optimal.

Second, the *efficient equilibrium* $\xi^{\mathrm{eff}} = (\xi_t^{\mathrm{eff}})_{t\geq 0}$ which maximizes utility of a fictitious world representative consumer and fully corrects the inefficiency of the Laissez faire solution. Along the efficient equilibrium, taxes are determined by the Pigouvian solution

$$\tau_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \Big(\phi_L + (1-\phi_L)\phi_0.(1-\phi)^n\Big) \sum_{\ell\in\mathbb{L}} \gamma^{\ell} Y_{t+n}^{\ell}. \tag{24}$$

The climate tax determined by (24) is called the efficient tax policy and is denoted $\tau^{\text{eff}} = (\tau_t^{\text{eff}})_{t \geq 0}$. If the efficient solution follows a balanced growth path on which output and consumption grow at constant and identical rate $g \geq 0$, (24) takes the simpler form

$$\tau_t^{\text{eff}} = \bar{\tau}^{\text{eff}} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^\ell, \quad \bar{\tau}^{\text{eff}} := \frac{\phi_L}{1 - \beta(1+g)^{1-\sigma}} + \phi_0 \frac{1 - \phi_L}{1 - \beta(1+g)^{1-\sigma}(1-\phi)}. \tag{25}$$

Thus, on a balanced growth path, the optimal tax is a constant share $\bar{\tau}^{\text{eff}}$ of world output weighted by the damage parameters $\gamma^\ell$.

As the aggregate equilibrium solution (23) does not specify disaggregated consumption in each region, it is independent of the transfer policy $\theta = (\theta^\ell)_{\ell \in \mathbb{L}}$. Once such a transfer policy is specified, the consumption vector $C_t = (C_t^\ell)_{\ell \in \mathbb{L}}$ and the supporting transfers $(T^\ell)_{\ell \in \mathbb{L}}$ can be determined by (16) and (9). The main advantage of determining an aggregate allocation first is that the equilibrium equations give rise to a forward-recursive structure which greatly simplifies their computation.

# 2 Solving the General Model

This section develops an algorithm to compute the aggregate equilibrium sequence $(\xi_t)_{t \geq 0}$ defined in (23) under alternative specifications of the climate tax policy $(\tau_t)_{t \geq 0}$. We confine attention to policies where the tax in period $t$ is determined endogenously as

$$\tau_t = \bar{\tau} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^\ell. \tag{26}$$

Specification (26) induces the Laissez faire equilibrium by setting $\bar{\tau} = 0$ and the (approximated) efficient solution for $\bar{\tau} = \bar{\tau}^{\text{eff}}$ defined as in (25).[2] One can also choose other values for $\bar{\tau}$ and it is also possible to specify the sequence $(\tau_t)_{t \geq 0}$ exogenously.

To reduce the number of equilibrium conditions, we first perform a few simple substitutions. First, substitute (10) using (11) and (12) into (13) and define $\phi_Z := \phi_L + (1-\phi_L)\phi_0$ to obtain climate damage in region $\ell$ as a function

$$D_t^\ell = \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1}) := 1 - \exp\left\{ -\gamma^\ell \left( \phi_Z \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell + S_{1,t-1} + (1-\phi) S_{2,t-1} - \bar{S} \right) \right\}. \tag{27}$$

Substituting (27) into (1) permits final output in region $\ell$ at time $t$ to be written as

$$Y_t^\ell = (1 - \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1})) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}). \tag{28}$$

[2] To evaluate the accuracy of this approximation in our simulations, we (a) verify that output and consumption both converge to a balanced growth path and (b) recompute optimal taxes based on the original formula (24) using the series of output and consumption from our simulation. The differences between both tax rates are sufficiently small, notably during the initial simulation periods when the economy still converges to the balanced path such that computing taxes based on (26) is fully justified.

Making the same substitution, the first order conditions (2) of the final sector become

$$
\begin{aligned}
(1-\hat{D}^\ell(\mathbf{X}_t,\mathbf{S}_{t-1}))Q^\ell_{0,t}\partial_K F_0(K^\ell_{0,t},N^\ell_{0,t},(E^\ell_{i,t})_{i\in\mathbb{I}}) &= r_t && \text{(29a)}\\
(1-\hat{D}^\ell(\mathbf{X}_t,\mathbf{S}_{t-1}))Q^\ell_{0,t}\partial_N F_0(K^\ell_{0,t},N^\ell_{0,t},(E^\ell_{i,t})_{i\in\mathbb{I}}) &= w^\ell_t && \text{(29b)}\\
(1-\hat{D}^\ell(\mathbf{X}_t,\mathbf{S}_{t-1}))Q^\ell_{0,t}\partial_{E_i} F_0(K^\ell_{0,t},N^\ell_{0,t},(E^\ell_{i,t})_{i\in\mathbb{I}}) &= p^\ell_{i,t} \quad \forall i\in\mathbb{I}. && \text{(29c)}
\end{aligned}
$$

In the following computations, it will be more convenient to use (28) and (29) instead of the original equations (1) and (2).

## 2.1 Simulation parameters

The simulation fixes the set of regions $\mathbb{L}$, energy sectors $\mathbb{I}$, and exhaustible resources $\mathbb{I}_x \subset \mathbb{I}$, assuming that $0 < L := |\mathbb{L}|$ and $1 \leq I_x := |\mathbb{I}_x| \leq I := |\mathbb{I}|$. Given the sectoral structure, one needs to specify the functional forms of the production functions $(F_i)_{i\in\mathbb{I}_0}$ and exogenous sequences for labor supply $(\mathbf{N}^S_t)_{t\geq 0}$ and productivity $(\mathbf{Q}_t)_{t\geq 0}$. Further, values respecting above's sign restrictions must be assigned to the parameters $(\beta,\sigma)$ describing consumer behavior, extraction costs $(c_i)_{i\in\mathbb{I}_x}$, climate parameters $(\phi,\phi_0,\phi_L)$, and damage parameters $(\gamma^\ell)_{\ell\in\mathbb{L}}$. Finally, we fix initial values for the climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$, stocks of global resources $(R_{i,0})_{i\in\mathbb{I}_x}$, and aggregate capital $\bar{K}_0 > 0$ and choose the value $\bar{\tau}$ for the climate tax policy (26).

## 2.2 Forward-recursive structure

Our numerical algorithm exploits the forward-recursive structure of the model to determine the vector $\xi_t$ defined in (23) as a function of $\xi_{t-1}$ and exogenous variables. To make this idea precise, partition the equilibrium vector as $\xi_t = (\xi^1_t, \xi^2_t)$ where

$$
\begin{aligned}
\xi^1_t &:= (\mathbf{Y}_t,\mathbf{E}_t,\mathbf{K}_t,\mathbf{N}_t,\mathbf{X}_t,r_t,\mathbf{w}_t,\mathbf{p}_t,\mathbf{v}_t,\tau_t) && \text{(30a)}\\
\xi^2_t &:= (\mathbf{S}_t,\bar{C}_t,\bar{K}_{t+1}). && \text{(30b)}
\end{aligned}
$$

Note that $\xi^1_t$ is an $M := 4L(I+1)+(L+1)I_x+2$-dimensional vector taking values in

$$
\Xi^1 := \mathbb{R}^L_{++}\times\mathbb{R}^{LI}_{++}\times\mathbb{R}^{L(I+1)}_{++}\times\mathbb{R}^{L(I+1)}_{++}\times\mathbb{R}^{LI_x}_{++}\times\mathbb{R}_{++}\times\mathbb{R}^L_{++}\times\mathbb{R}^{LI}_{++}\times\mathbb{V}\times\mathbb{R}_+ \qquad (31)
$$

where $\mathbb{V} := \prod_{i\in\mathbb{I}_x}[c_i,\infty[\subset \mathbb{R}^{I_x}_+$. Vector $\xi^2_t$ takes values in the set $\Xi^2 := \mathbb{R}^2\times\mathbb{R}_{++}\times\mathbb{R}_{++}$. For each period $t \geq 0$, the relevant pre-determined variables are collected in a vector

$$
\theta_t := (\mathbf{N}^s_t,\mathbf{Q}_t,\mathbf{v}_{t-1},\mathbf{S}_{t-1},\bar{C}_{t-1},\bar{K}_t) \qquad (32)
$$

with values in $\Theta := \mathbb{R}^L_{++}\times\mathbb{R}^{L(I+1)}_{++}\times\mathbb{V}\times\mathbb{R}^2\times\mathbb{R}_{++}\times\mathbb{R}_{++}$. Note that $\theta_t$ consist of the exogenous variables $(\mathbf{N}^s_t,\mathbf{Q}_t)$ and a number of endogenous variables from $\xi_{t-1}$.

Given $\theta_t \in \Theta$, the main step in our algorithm is to determine $\xi_t^1$ by simultaneously solving equations (3), (4), (5), (6), (7), (19), (20), (26), (28), and (29). Note that these conditions constitute a system of $LI_x + 3LI_x + L(I - I_x) + 2L(I - I_x) + I_x + L + 1 + 1 + L + L(I + 2) = 4L(I + 1) + (L + 1)I_x + 2 = M$ non-linear equations that can potentially be solved to obtain a unique vector $\xi_t^1 \in \Xi^1$.

To formalize this problem, define the mapping $\Phi : \Xi^1 \times \Theta \longrightarrow \mathbb{R}^M$ such that given $\theta_t \in \Theta$, $\xi_t^1$ solves equations (3)-(7), (19), (20), (26), (28), and (29) if and only if $\Phi(\xi_t^1, \theta_t) = 0$. For example, if $1 \in \mathbb{I}_x$, the first component function $\Phi_1 : \Xi^1 \times \Theta \longrightarrow \mathbb{R}$ defined by the first entry of equation (3) would be $\Phi_1(\xi_t^1, \theta_t) = E_{1,t}^1 - Q_{1,t}^1 F_1(K_{1,t}^1, N_{1,t}^1, X_{1,t}^1)$.

Given the pre-determined variables $\theta_t$ and the solution $\xi_t^1$, $\xi_t^2$ can be determined directly by equations (11), (17), and (22). These equations define a function $\Psi : \Xi^1 \times \Theta \longrightarrow \Xi^2$ which determines $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$. Determining $\xi_t^1$ and $\xi_t^2$ in this fashion based on predetermined variables collected in $\theta_t$ defines one iteration step of our model.

## 2.3 Computational algorithm

The following sequential structure illustrates our computational algorithm for an iteration of the model of length $t^{\max} > 0$.

**Step 1**: Initialization for $t = 0$:[3]

(a) Choose candidate values for initial consumption $\bar{C}_{-1} > 0$ and initial resource prices $\mathbf{v}_{-1} = (v_{i,-1})_{i \in \mathbb{I}_x} \in \mathbb{V}$. If $R_{i,0} = \infty$, set $v_{i,-1} = c_i$, otherwise $v_{i,-1} > c_i$.

(b) Use these values together with $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ and $\bar{K}_0 > 0$ to determine the endogenous part of $\theta_0$. Set $t = 0$.

**Step 2:** Iteration for $0 \leq t \leq t^{\max}$:

(a) Compute $\theta_t$ using $(\mathbf{N}_t^S, \mathbf{Q}_t)$ and the relevant endogenous variables from $t - 1$.

(b) Compute $\xi_t^1$ by solving $\Phi(\xi_t^1, \theta_t) = 0$ as outlined above.

(c) Compute $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$ as outlined above and check the following conditions:

- If $\bar{K}_{t+1} < 0$, return to **Step 1** and decrease $\bar{C}_{-1}$.
- If $\bar{C}_t < \bar{C}_t^{\text{crit}}$, return to **Step 1** and increase $\bar{C}_{-1}$.
- Otherwise, increase $t$ by 1.

**Step 3:** Verification of resource constraints in $t = t^{\max}$:

[3]Specifying initial values for $\mathbf{v}_{-1}$ and $\bar{C}_{-1}$ and computing $\mathbf{v}_0$ and $\bar{C}_0$ using (7) and (17) allows us to cast all computations for $t = 0$ in the same form as for periods $t > 0$. The structure of equations (7) and (17) and the fact that the values $\mathbf{v}_{-1}$ and $\bar{C}_{-1}$ only affect $\mathbf{v}_0$ and $\bar{C}_0$ shows that this approach is mathematically equivalent to assigning initial values to $\mathbf{v}_0$ and $\bar{C}_0$ directly.

(a) For all $i \in \mathbb{I}_x$, compute $R_{i,t^{\max}+1} := R_{i,0} - \sum_{t=0}^{t^{\max}} \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell}$:

- If $R_{i,t^{\max}+1} < 0$, return to **Step 1** and increase $v_{-1,i}$.
- If $R_{i,t^{\max}+1} > R_i^{\text{crit}}$, return to **Step 1** and increase $v_{-1,i}$.

(b) If $0 < R_{i,t^{\max}+1} < R_i^{\text{crit}}$ for all $i \in \mathbb{I}_x$, complete the iteration. ■

Step 2(c) in the previous algorithm requires the specification of a (typically time-dependent) lower bound $\bar{C}_t^{\text{crit}}$ for consumption in period $t$.[4] The condition $\bar{C}_t > \bar{C}_t^{\text{crit}}$ for all $t$ serves to exclude cases where consumption *implodes*, i.e., converges to zero. This case occurs when initial consumption $\bar{C}_{-1}$ is chosen too small. Conversely, if $\bar{C}_{-1}$ is chosen to large, consumption *explodes*, i.e., grows too fast relative to output. In this case, the condition $\bar{K}_{t+1} > 0$ for all $t$ will eventually be violated. Excluding both cases determines a unique initial value $\bar{C}_{-1}$ for which the equilibrium dynamics are well defined and satisfy the transversality condition (18). These features are well-known for the neoclassical growth model in state space form which exhibits saddle-path stability requiring initial consumption to be chosen on the stable manifold of values which converge to the steady state. These features carry over to the present more complicated model. Our numerical approach determines the unique sustainable initial level $\bar{C}_{-1}$ such that $\bar{K}_{t+1} > 0$ and $\bar{C}_t > \bar{C}_t^{\text{crit}}$ for all $t \le t^{\max} + N^{\text{ahead}}$ for some $N^{\text{ahead}} \ge 0$.[5]

The conditions evaluated in Step 3 concern the world resource constraints (21). Clearly, this condition becomes relevant only for resources $i \in \mathbb{I}_x$ for which $R_{i,0} < \infty$. Suppose this is the case and define $R_{i,t+1} := R_{i,t} - \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell}$ as the world resource stock at the end of period $t$. For any candidate resource price $\hat{v}_{i,-1}$, the induced sequence $(\hat{R}_{i,t})_{t\ge 0}$ of world resource stocks is strictly decreasing and, therefore, converges to a unique limit $\hat{R}_{i,\infty}$ which is zero at equilibrium. In our simulations, we establish that the sequence $(\hat{R}_{i,t})_{t\ge 0}$ becomes approximately constant within the length of iteration such that $\hat{R}_{i,\infty}$ can be approximated by $\hat{R}_{i,t^{\max}+1}$. We now adjust the initial resource price $\hat{v}_{i,-1}$ until $\hat{R}_{i,t^{\max}+1}$ becomes approximately zero, increasing $\hat{v}_{i,-1}$ when $\hat{R}_{i,t^{\max}+1} < 0$ and decreasing $\hat{v}_{i,-1}$ when $\hat{R}_{i,t^{\max}+1} < 0$. The iteration stops when all terminal resource stocks are (in absolute terms) less than a pre-specified critical value $R_i^{\text{crit}}$ which is chosen close to zero. The current value $\hat{v}_{i,-1}$ then approximates the initial equilibrium resource price $v_{i,-1}$.

[4] Our simulations use $\bar{C}_t^{\text{crit}} = \bar{c}^{\text{crit}} \left(\sum_{\ell\in\mathbb{L}} Y_t^{\ell} - \sum_{i\in\mathbb{I}_x} c_i \sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell}\right)$ where $\bar{c}^{\text{crit}}$ is a small number.

[5] In fact, to reduce computation time, we choose initial consumption $C_{-1}$ such that $\bar{C}_t > \bar{C}_t^{\text{crit}}$ and $\bar{K}_{t+1} > 0$ holds for all $0 \le t \le N^{\text{ahead}} = 10$. Then, in each future period $t > 0$, the value $\bar{C}_t$ delivered by the Euler equation (17) is (slightly) adjusted such that $\bar{C}_{t+n} > \bar{C}_{t+n}^{\text{crit}}$ and $\bar{K}_{t+n} > 0$ holds for all $0 \le n \le N^{\text{ahead}}$. Thus, in each period, we adjust consumption to ensure that the consumption-capital dynamics is stable over the next $N^{\text{ahead}}$ periods. As these adjustments are small if $N^{\text{ahead}}$ is chosen sufficiently large, our approach is equivalent to choosing initial consumption $C_{-1}$ such that the dynamics is stable for all $t \le t^{\max} + N^{\text{ahead}}$ but turned out to be computationally faster. In addition, one can successively increase the accuracy of the simulations by gradually increasing $N^{\text{ahead}}$.

## 2.4 Computational details

The key challenge in our algorithm is to determine the vector $\xi_t^1$ which solves the condition $\Phi(\xi_t^1, \theta_t) = 0$ in Step 2(b). Mathematically, this is a standard fixed point problem which can be solved using standard numerical routines like the Newton-Raphson algorithm, etc. As our simulations are directly implemented in $C++$, however, we designed our own more 'economic' algorithm. Intuitively, this algorithm successively equates marginal products across different markets by reallocating production factors based on the differences between their respective marginal products. We employ a nested market structure where the labor allocation in both regions is determined first for a given allocation of capital and exhaustible resources, then the global capital allocation is determined for a given resource allocation (with labor constantly readjusting) and, finally, the global resource allocation is computed. While potentially inefficient in terms of computational speed, this proved to be a reliable way of computing the solution $\xi_t^1$. Details are provided in Appendix A.

## 2.5 Regional consumption and transfers

The aggregate equilibrium (23) computed in the previous sections does not specify regional consumption $\mathbf{C}_t = (C_t^\ell)_{\ell\in\mathbb{L}}$ and the transfers (9) between regions. Computation of these values requires the specification of a transfer policy $\theta = (\theta^\ell)_{\ell\in\mathbb{L}}$ and the initial distribution of capital $(K_0^\ell)_{\ell\in\mathbb{L}}$ and exhaustible resources $(R_{i,0}^\ell)_{\ell\in\mathbb{L}}$ of each type $i \in \mathbb{I}_x$. Once these objects are specified, we need to approximate lifetime labor incomes $(W^\ell)_{\ell\in\mathbb{L}}$ and transfer incomes $(T^\ell)_{\ell\in\mathbb{L}}$ defined as above. For each $\ell \in \mathbb{L}$ define for $N > 0$

$$W_N^\ell := \sum_{t=0}^{N} q_t w_t^\ell N_t^\ell \tag{33}$$

and total discounted tax revenue

$$T_N := \sum_{t=0}^{N} q_t \tau_t \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^\ell. \tag{34}$$

Both sequences $(W_N^\ell)_{N\geq 0}$ and $(T_N)_{N\geq 0}$ are strictly increasing and we verify numerically that they converge sufficiently fast and become nearly constant as $N \to t^{\max}$ where we use $t_{\max} = 50$ in our simulations. This allows us to approximate $W^\ell$ by $\hat{W}^\ell := W_{t^{\max}}^\ell$ and $T^\ell$ by $\hat{T}^\ell := \theta^\ell T_{t^{\max}}$.

With these approximations, one can employ (16) to obtain (approximated) consumption in region $\ell \in \mathbb{L}$ as

$$\hat{C}_t^\ell = \hat{\mu}^\ell \bar{C}_t = \frac{r_0 K_0^\ell + \hat{W}^\ell + \Pi^\ell + \hat{T}^\ell}{\sum_{k\in\mathbb{L}} \left( r_0 K_0^k + \hat{W}^k + \Pi^k + \hat{T}^k \right)} \bar{C}_t \quad \forall t \geq 0 \tag{35}$$

with profit incomes $\Pi^\ell$ determined by (14) from the initial distribution of exhaustible resources across regions.

# 3 Calibrating the Model

The world economy is divided into $L = 2$ regions. Region $\ell = 1$ represents the member states of the *'Organization for Economic Cooperation and Development'* and will be called the *OECD countries*. Region $\ell = 2$ comprises the rest of the world and will be referred to as the *NOECD countries*. Our study compares the Laissez faire and the optimal tax policy discussed in the previous sections and determines optimal transfers between OECD and NOECD countries for the transfer policy given in equation (9).

## 3.1 Main calibration targets

Following GHKT and Acemoglu et al. (2012), each model period represents ten years. The initial simulation period ends in $t = 2015$ and is called the baseline period. Our calibration is based on the following empirical observations which we match in the baseline period:

- The world population share of OECD-countries is currently 18%[6]
- GDP in OECD countries makes up 69% of current world GDP[7]
- OECD countries owned 68,5% of the global capital stock in 2015[8]
- 17% of global crude oil and 10% of natural gas reserves are located in OECD countries.[9]

Further targets which we match in our simulations are introduced below.

[6]The World Bank. World Development Indicators (2015). *Total Population.* Available from `http://data.worldbank.org/indicator/SP.POP.TOTL`

[7]The World Bank. World Development Indicators (2015). *GDP(current US-\$).* Retrieved from `http://data.worldbank.org/indicator/NY.GDP.MKTP.CD`

[8]Berlemann & Wesselhoeft (2014) reported estimates of capital stocks for 103 countries for the period 1970-2011 using the perpetual inventory method. Their estimates imply a world capital stock of 64,499 Billion US-$\$_{2000}$ of which 44,208 Billion US-$\$_{2000}$ (68,5%) is located in OECD-member states.

[9]According to the German Federal Institute for Geosciences and Natural Resources (BGR), current global crude oil reserves are 219 Gt (Giga tons) of which 181 Gt are located in NOECD-countries. Current natural gas reserves are 197,8 Trn.$m^3$, of which 178 Trn.$m^3$ are in NOECD-countries.

## 3.2 Energy sectors

Each region has $I = 3$ energy sectors. Sector $i = 1$ comprises energy outputs and services derived from *crude oil and natural gas* including all traffic and transportation services based on oil and gas such as motorvehicles, cargo aircrafts, railroad cargo etc. as well as oil refineries which produce petroleum products. Sector $i = 2$ produces energy based on *coal* (anthracite coal and lignite). Its output comprises essentially coal-based power generation and heat. Sector $i = 3$ subsumes all energy goods and services which do not generate emissions. For simplicity, we assume that production in this sector is exclusively based on renewable energy sources.[10] With our previous notation we thus have $\mathbb{I} = \{1, 2, 3\}$ and $\mathbb{I}_x = \{1, 2\}$.

The production technologies in (1),(3), and (5) are specified as follows. The production function in the final sector takes the CES-form

$$F_0(K, N, E) = \Big[\alpha_{0,K} K^{\varrho_0} + \alpha_{0,N} N^{\varrho_0} + (1 - \alpha_{0,K} - \alpha_{0,N}) G((E_i)_{i \in \mathbb{I}})^{\varrho_0}\Big]^{\frac{1}{\varrho_0}} \tag{36}$$

where $\alpha_{0,K} > 0$, $\alpha_{0,N} > 0$, $\alpha_{0,K} + \alpha_{0,N} < 1$, and $\varrho_0 < 1$.

Exhaustible energy sectors $i \in \mathbb{I}_x = \{1, 2\}$ use a technology of the form

$$F_i(K, N, X) = X^{\alpha_{i,X}} \Big[\alpha_{i,K} K^{\varrho_i} + (1 - \alpha_{i,K}) N^{\varrho_i}\Big]^{\frac{1-\alpha_{i,X}}{\varrho_i}} \tag{37}$$

where $0 < \alpha_{i,X} < 1$, $0 < \alpha_{i,K} < 1$, and $\varrho_i < 1$. This specification allows us to vary the elasticity of substitution $\frac{1}{1-\varrho_i}$ between labor and capital while maintaining our earlier assumption that exhaustible resources constitute an essential input to production.

Finally, the technology used by the clean sector $i = 3$ is again of the CES-type

$$F_3(K, N) = \Big[\alpha_{3,K} K^{\varrho_3} + (1 - \alpha_{3,K}) N^{\varrho_3}\Big]^{\frac{1}{\varrho_3}} \tag{38}$$

where $0 < \alpha_{3,K} < 1$ and $\varrho_3 < 1$.

In our benchmark parametrization we set $\varrho_i = 0$ in (36), (37), and (38) inducing a Cobb-Douglas technology in each sector which allows us calibrate the $\alpha$ parameters based on observed cost shares of production factors.

With the previous interpretation in mind, we set $\alpha_{0,K} = 0.3$ in (36) which is a value commonly used in the literature. We used world input-output tables constructed in Timmer et al. (2015) to calculate the cost share of energy in final output production and accordingly set $\alpha_{0,E} = 0.075$ implying a cost share of labor equal to 62.5%.[11]

[10] As nuclear energy production would also be included in sector 3, this abstracts from the fact that uranium is an exhaustible resource, too. This seems justified, however, because the existing stocks of uranium are abundant relative to fossil reserves.

[11] The cost share of energy is higher than the value of 4% used in GHKT. However, as energy inputs to final production represent intermediate goods and services produced from fossil resources in our model rather than exhaustible resources as in GHKT, our higher share of energy costs reflects the value added at the energy production stage.

For sector $i = 1$, we computed cost shares corresponding to $\alpha_{1,K} = 0.85$ and $\alpha_{1,X} = 0.33$ based on data from the U.S Bureau of Economic Analysis (2007).[12]

As sectors $i = 2, 3$ mainly produce electricity, we can base our parameter choices on the nominal electricity generation costs for Germany reported in Hillebrand (1997).[13] For coal-fired power plants, this study delivers production elasticities $\alpha_{2,K} = 0.69$ for capital and $\alpha_{2,X} = 0.26$ for coal, respectively, which we use directly for sector $i = 2$.

It seems more difficult to choose these parameters for sector $i = 3$ which comprises all emissions-free technologies including nuclear power generation. The share of capital costs for nuclear power plants in Hillebrand (1997) is $\alpha_{3,K} = 0.7$. While nuclear energy makes up a large part of emissions free energy production, sector $i = 3$ also includes renewable energies like wind or solar power for which Loeschel & Otto (2009) report an even higher share of capital costs. For this reason, we choose a slightly higher capital share setting $\alpha_{3,K} = 0.75$. The previous two values are also in line with the general observation made by the Department of Energy & Climate Change (2013) that electricity generated from nuclear as well as wind and hydro power plants is relatively more capital intensive compared to conventional fossil-based electricity or thermal power generation.

Energy goods and services used in final output production are aggregated in two steps. First, the following function $G_2$ aggregates outputs produced in sectors $i = 2$ and 3 (electricity and heat) to an intermediate composite

$$EL_t^\ell = G_2(E_{2,t}^\ell, E_{3,t}^\ell) := \left[\kappa_2 (E_{2,t}^\ell)^{\rho_2^E} + (1-\kappa_2)(E_{3,t}^\ell)^{\rho_2^E}\right]^{\frac{1}{\rho_2^E}}. \tag{39}$$

The parameter $\rho_2^E$ determines the elasticity of substitution between $CO_2$-intensive and clean electricity. Setting $\rho_2^E = 0.6$ in (39) we follow Loeschel & Otto (2009) who report an elasticity of substitution equal to 2.5. We also let $\kappa_2 = 0.5$ which is in line with the observations in GHKT who choose a relative price between dirty and clean electricity generation equal to unity.

A second function $G_1$ then aggregates the electricity composite $EL_t^\ell$ with oil and gas-based energy services $E_{1,t}^\ell$ produced in sector $i = 1$ to the final energy composite

$$E_t^\ell = G_1(E_{1,t}^\ell, EL_t^\ell) := \left[\kappa_1 (E_{1,t}^\ell)^{\rho_1^E} + (1-\kappa_1)(EL_t^\ell)^{\rho_1^E}\right]^{\frac{1}{\rho_1^E}}. \tag{40}$$

[12] The data highlights inter-sectoral linkage for 389 industries/commodities for the United States that can be aggregated to specific sectors which are relevant for this study, for instance "Refineries" or "Transportation". Especially for these three isolated industry groups, which represent our energy type $i = 1$ "oil and gas based energy goods and services", we then calculated the parameters representing the cost shares for capital, labor and resources. Additional details are available upon request.

[13] Drawing on data sources for different countries we presume that the underlying technologies are similar enough such that the resulting cost shares for capital and labor are roughly the same.

Using industry data from the U.S Bureau of Economic Analysis (2007), we set $\kappa_1 = 0.3818$ in (40), which corresponds to the cost of electricity and heat production relative to the cost of transportation per unit of GDP.

There is some considerable degree of freedom to restrict the parameter $\rho_1^E$ in (40) which determines the elasticity of substitution between electricity and fossil fuel.[14] We choose a moderately positive value of $\rho_1^E = 0.2$. This also ensures that energy produced by sector $i = 1$ is not an essential input to final production which allows for the model to have a well-defined balanced growth path.

## 3.3 Labor supply and productivity growth

The initial distribution of labor supply $\mathbf{N}_0^s = (N_0^\ell)_{\ell\in\mathbb{L}}$ is chosen as $N_0^1 = 0.18$ and $N_0^2 = 0.82$ based on relative population sizes of the two regions with world labor supply normalized to unity. Growth in our model enters via exogenous labor-augmenting change due to which the sequence $(\mathbf{N}_t^s)_{t\geq 0}$ grows at constant rate $g > 0$ in each component. Setting $g = 0.16$ implies an annual growth rate of productivity equal to 1.5% which is a conservative estimate in line with GHKT and most studies of the business cycle.

Differences in productivity are captured by region-specific total factor productivities $Q_{i,t}^\ell \equiv Q_i^\ell$ which are constant over time but allowed to differ across regions $\ell \in \mathbb{L}$ and sectors $i \in \mathbb{I}_0$. For the final sector $i = 0$, we chose relative productivities to match the observed GDP shares reported above while their absolute levels induce a plausible world output of about 700 trillion \$ in the initial modelling period which is also used in GHKT. For energy sectors $i \in \mathbb{I}$, the relative sizes of productivities are chosen to obtain a plausible energy mix in both regions along the Laissez faire equilibrium.[15]

## 3.4 Resource sectors

Global extraction costs of coal reported by the International Energy Agency (2010, p. 212) average to 43 \$ per ton of coal. This value corresponds to a parameter choice $c_2 = 0.000043$ in our model which we choose directly in our simulations.

For oil and gas, however, empirically measured extraction costs differ considerably across regions and, in addition, often represent short term operating costs not including long-term costs for capital, etc.[16] For this reason, our calibration strategy is to determine

[14] A recent study by Stern (2012) reports values for this elasticity ranging from −3.265 to 8.922.

[15] Data from the International Energy Agency (2014) report that total primary energy supply in OECD countries decomposes into a share of 62% for oil and natural gas, 18% for coal, and 20% for clean energies. For NOECD countries, the corresponding shares are 46%, 35%, and 18% which we match in our baseline period.

[16] For instance, short term operating costs of crude oil extraction reported in the World Economic Outlook 2015 by the International Monetary Fund (2015) range from 4.4 \$/bbl (31.4 \$/t) in Kuwait

the cost parameter $c_1$ such that the induced initial price $v_{1,0}$ of the composite oil/gas resource at the Laissez faire equilibrium is consistent with empirically observed prices. Using data from the World Bank (2015a) from 2002-2016, we compute the average price for crude oil and natural gas weighted by their respective shares of total global reserves. This gives an initial price of 49.8 \$/bbl corresponding to 356 \$/t which we match in our initial simulation period by setting $c_1 = 0.00025568$.[17]

According to data from the Federal Institute for Geosciences and Natural Resources (2015), global fossil resources include 219 Gt of crude oil and 179 Gt of natural gas.[18] In accordance with the stylized facts reported above, we fix the initial stock of resource 1 (oil/gas) in OECD countries at $R^1_{1,0} = 56$ Gt and $R^2_{1,0} = 342$ Gt in NOECD countries. For resource 2 (coal), we adopt the same arguments as in GHKT to assume that there is no scarcity rent on coal such that the stock of coal is not exploited. Formally, this corresponds to $R_{2,0} = \infty$ in our simulations which can be justified by a backstop technology which will replace current coal usage in the future. This restriction also implies $v_{2,t} = c_2$ for all $t \geq 0$. As coal extraction generates zero profits, the world distribution of coal reserves is irrelevant. A modification of these restrictions is explored in Section 5 where there is no backstop technology implying that the initial stock of coal is finite.

### 3.5 Climate dynamics and damages

As our model of the Carbon cycle (11) and the damage function (13) are identical to GHKT, we also use their parameter values setting $\phi_0 = 0.393$, $\phi_L = 0.2$, $\phi = 0.0228$, and $\gamma_1 = \gamma_2 = 5.3 \times 10^{-5}$ in the benchmark case with homogeneous climate damages. Regional differences in $\gamma^\ell$ will be explored below. The pre-industrial $CO_2$-level is $\bar{S} = 581$ and the initial values for permanent and non-permanent $CO_2$ are $S_{1,-1} = 705$ and $S_{2,-1} = 123$ GtC. For these values, global carbon concentration in the initial simulation period matches the empirically observed $CO_2$-concentration of 853 GtC in 2015 obtained from the Earth System Research Laboratory (2016).

The carbon content $\zeta_i$ of resources $i \in \mathbb{I}_x$ are specified as follows. For $i = 1$, we average the emission factors of crude oil and natural gas weighted by the respective shares of global reserves. This yields a specific carbon content of $\zeta_1 = 0.74681$, corresponding to 746.8 KgC/t. For $i = 2$, we average the emission factors of lignite and anthracite weighted by the respective shares of total global coal reserves. This gives a specific emission content of $\zeta_2 = 0.55854$ corresponding to 558.5 KgC/t.[19]

up to 12 \$/bbl (85.7 \$/t)in Venezuela.

[17] The equal weighting of both prices

[18] The figures are based on the concept of 'proven reserves' which allows for changes in resource prices but assumes that firms can fully exploit these resources without any change in the applied technology.

[19] Our specific carbon content of 746.8 KgC/t oil and gas, is slightly lower than a carbon content of 844 KgC/t oil given in GHKT. This is due to our additional consideration of natural gas which has a

## 3.6 Consumption sector

Restricting consumer utility as in (15), we choose $\sigma = 1$ which gives a logarithmic utility function. The annual discount factor is $\beta = 0.985$, so for the model we set $\beta = 0.985^{10}$. These values are identical to the ones used by GHKT in their benchmark scenario. Further, in accordance with the stylized facts reported above, consumers in region 1 own 68.5% of the initial world capital stock. The latter is chosen to obtain a capital-to labor ratio close to its long-run value along the Laissez-faire equilibrium. This avoids a transitory effect due to initial capital adjustment dynamics.

Table 1 summarizes the parameters choices motivated above which are used in the benchmark simulation.

| Simulation parameters | | | |
|---|---|---|---|
| Final sector | | | |
| $\rho_0 = 0$ | $\alpha_{0,K} = 0.3$ | $\alpha_{0,N} = 0.625$ | $\alpha_{0,E} = 0.075$ |
| $\rho_1^E = 0.2$ | $\rho_2^E = 0.6$ | $\kappa_1 = 0.3818$ | $\kappa_2 = 0.5$ |
| $Q_0^1 = 3.1$ | $Q_0^2 = 0.7$ | | |
| Energy sectors | | | |
| $\rho_1 = 0$ | $\alpha_{1,K} = 0.85$ | $\alpha_{1,X} = 0.33$ | $Q_1^1 = 96,\ Q_1^2 = 19$ |
| $\rho_2 = 0$ | $\alpha_{2,K} = 0.69$ | $\alpha_{2,X} = 0.26$ | $Q_2^1 = 3.5, Q_2^2 = 8.2$ |
| $\rho_3 = 0$ | $\alpha_{3,K} = 0.75$ | | $Q_3^1 = 40,\ Q_3^2 = 50$ |
| Resource sectors | | | |
| $c_1 = 0.000225568$ | $c_2 = 0.000043$ | | |
| Climate parameters | | | |
| $\zeta_1 = 0.74681$ | $\zeta_2 = 0.55854$ | $\bar{S} = 581$ | $\phi_L = 0.2$ |
| $\phi_0 = 0.393$ | $\phi = 0.0228$ | $\gamma_1 = 0.000053$ | $\gamma_2 = 0.000053$ |
| Consumption sector | | | |
| $\beta = 0.985^{10}$ | $\sigma = 1$ | | |
| Initial values | | | |
| $K_0 = 0.15$ | $R_{1,0} = 398$ | $S_{1,-1} = 705$ | $S_{2,-1} = 123$ |

Table 1: Parameter values used in benchmark simulation.

# 4 Simulation results

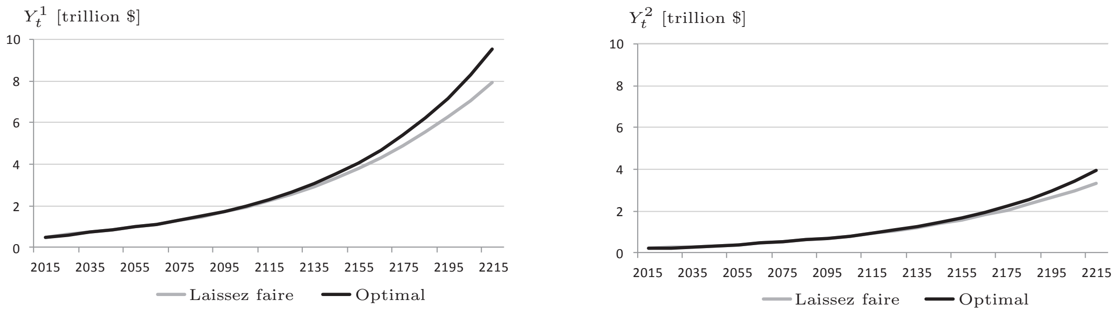
Using the parametrization listed in Table 1, the simulation results presented in this section compare the optimal and laissez faire equilibrium at the final stage, the energy stage, the resource stage, and the climate stage. We also study the direction and size of

lower specific carbon content compared to oil. This small deviation holds also in the case of coal, since we choose a weighted average of anthracite and lignite with corresponding different carbon contents, while GHKT set their carbon content of coal equal to the value of anthracite. Expressed in carbon units, we assume 560 KgC/t of coal, while GHKT assume 716 KgC/t coal.

optimal transfers between the two regions based on the Pareto-improving transfer policy described above. There, we also allow for heterogeneities in climate damages which may be more severe in NOECD (i.e. low-income) countries.

## 4.1 Final output stage

Figure 1 compares production output $Y_t^\ell$ in both regions under Laissez faire and optimal taxation. For the initial period, our model predicts a world GDP $Y_t = Y_t^1 + Y_t^2$ of 707




(a) OECD

(b) NOECD

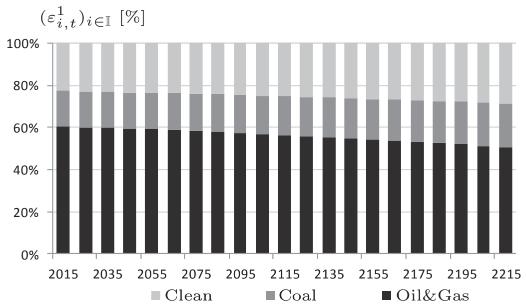
Figure 1: Final production in each region: Optimal vs. Laissez faire solution.

trillion US$ at the Laissez faire solution and 702 trillion under optimal taxation which closely matches the empirical value reported above.

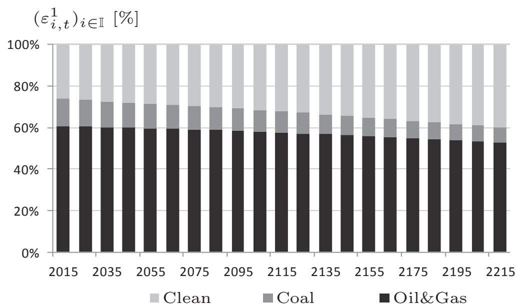
Under Laissez faire, output in both regions is slightly higher in the initial modelling periods until $t = 2055$. Quantitatively, the climate tax reduces GDP in OECD countries by about -0.5% in $t = 2015$ and 2025 and by about $-0.6\%$ in $t = 2035$ and 2045 relative to Laissez faire. For NOECD countries, output drops by about -0.9% in $t = 2015$ and 2025 and by -1.2% in $t = 2035$ and 2045. World output decreases by about -0.6% in t=2015 and 2025 and -0.8% in t=2035 and 2045 relative to the Laissez faire solution. From $t = 2055$ on, however, this effect reverses and output in both regions continues to grow under optimal taxation. After 100 years, world output along the optimal solution is already 2% higher than without taxation. This gap widens as these effects continue to grow, resulting in world GDP along the optimal solution almost 20% higher in $t = 2215$ relative to the Laissez faire. Summarizing, these results support the intuition that climate policies come at initial costs which are however negligible to the gains in the long-run.

## 4.2 Energy stage

We employ two measures to quantify sectoral changes at the energy level. The first one is the *energy mix* which measures the value share each energy sector contributes to the total value of energy production (the latter measured in units of final output). Figures 2 and 3 compare the energy mix in OECD and NOECD countries, respectively under Laissez faire and optimal taxation.



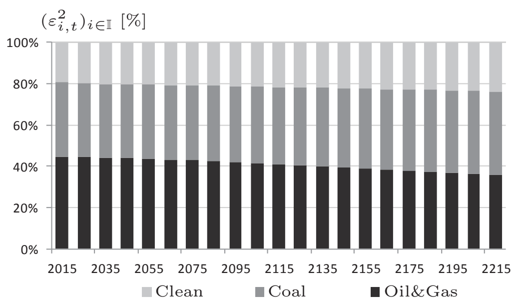

(a) Laissez faire



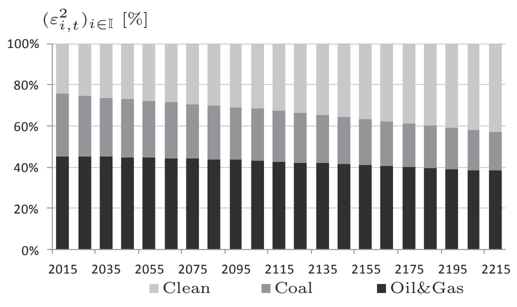

(b) Optimal taxation

Figure 2: Energy mix in OECD countries.

First, in accordance with empirical observations, fossil fuels in our model dominate energy usage in both regions under laissez faire: in the baseline period, emission free technologies amount to only 22% (19%) in the OECD (NOECD). Although economic activity in both regions highly depends on oil and gas, this dependence is much more severe in the OECD (while oil and gas constitute 77% of total fossil fuel use here, this share amounts to only 54% in NOECD countries). Conversely, NOECD countries comparatively stronger base their energy production on coal (36% out of total energy production in NOECD, 18% in OECD).




(a) Laissez faire



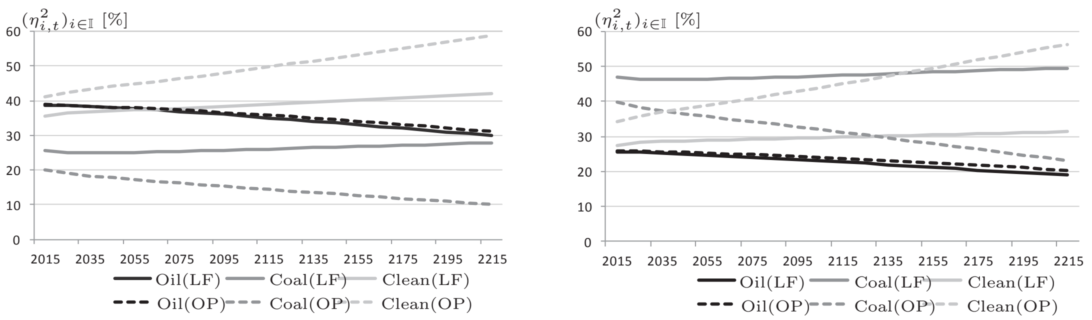

(b) Optimal taxation

Figure 3: Energy mix in NOECD countries.

Second, OECD countries are responsible for roughly 70% of global energy usage. This share is equivalent to OECD's share of global GPD reported above and is a direct consequence of our assumption that firms in both regions produce final output using an identical Cobb-Douglas production technology.

Over the next 200 years, our model predicts a gradual decline of oil energy shares, because oil resources become increasingly scarce over time. Oil use is gradually replaced by coal and clean energy technologies. Thus clean energies acquire a higher share over time even under Laissez faire.

The energy mix and regional shares of global energy use in the baseline period are roughly the same under the optimal policy. Over time, the introduction of a carbon tax leads to substitution of oil and coal based energy production technologies by clean technologies: 100 years from now, clean energy technologies produce 32% (33%) of total energy supply in the OECD (NOECD), which is an increase of 23% (34%) compared to baseline period values. Interestingly, the value share of oil under optimal policy is larger compared to the corresponding value under laissez faire in both regions considered here. The decarbonization of energy supply under optimal policy is therefore primarily driven by a reduction in coal based energy supply.




(a) OECD

(b) NOECD

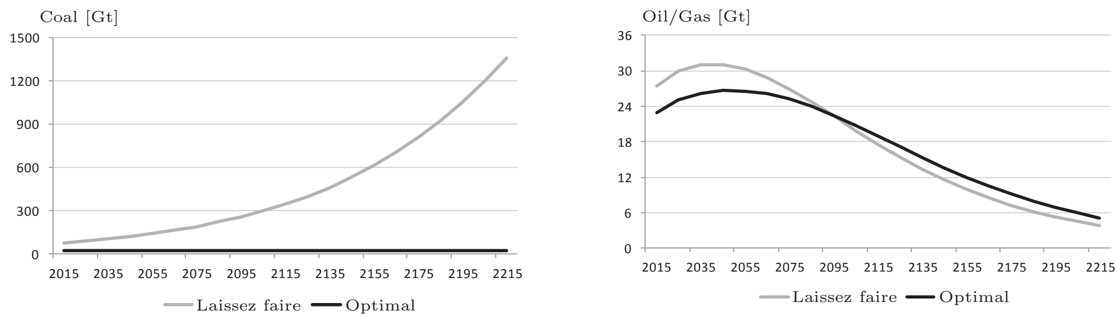
Figure 4: Employment shares in energy sectors: Optimal vs. Laissez faire solution.

A second measure of climate policy induced structural change are the employment effects in energy sectors, here measured by employment shares in sector $i$ out of total employment in energy sectors. Energy firms located in OECD countries employ only 16% of persons working in the energy stage around the world. This follows from regional differences in the labor force observed, where 18% of all people live in OECD countries, while 82% of the people live in NOECD countries.

Figure 4 shows employment shares under laissez faire (solid lines) and optimal policy (dashed lines). Under laissez faire, most people employed in NOECD energy production, work in the coal sector (47%). This share is only 25% in the OECD. Over time, employment shares of coal based energy production slightly increase in both regions. Even under laissez faire, structural change increases the fraction of people working in clean energy production over time. 100 years from now, 27% (32%) of all people employed inthe energy sector, work in clean energy production. Conversely, oil/gas shares decline from 38% (25%) in the OECD (NOECD) in 2015 to 30% (18%) in 2215. Comparing laissez faire and optimal policy outcomes, we find that employment shares in the oil sector are largely unaffected by taxation. Policy induced intrasectoral structural change is essentially characterized by a shift of employment from coal to clean energy production.

## 4.3 Resource stage

Figure 5 shows the extraction paths of coal and oil/gas in the laissez-faire and the optimal equilibrium. The model predicts an annual extraction of 7.3 Gt coal in the baseline




(a) Coal extraction

(b) Oil/Gas extraction

Figure 5: Global extraction of exhaustible resources: Optimal vs. Laissez faire solution.

period under laissez faire, which is close to empirical observations of 7.2 Gt reported by the U.S. Energy Information Administration 2014 (EIA). In the laissez faire equilibrium, coal extraction grows exponentially over the entire time interval displayed. Consequently coal must have a scarcity rent some time in the future, unless a backstop technology arrives. The emergence of this backstop in the future leads to current exploration pattern as if coal resources are infinite. Since coal extractions predicted by the model match empirical observations almost perfectly, this might suggest that the global coal market has already priced in the future adoption of a backstop technology.

Current estimates of global coal reserves reported by the EIA, are approximately 890 Gt. Under laissez faire, coal reserves would be exhausted completely within the next 60 years (2075). Hence, the emergence of a backstop must occur not later than 60 years from now. This is in contrast to GHKT, where the backstop has to appear some time within the next 200 years.

The introduction of a tax on fossil fuel in the baseline period reduces coal extraction instantaneously by 69%. Over the next 200 years, the extraction of coal remains almost constant at 2.1 Gt p.a. under optimal policy. This almost flat extraction path is due to an increasing carbon tax over time: while net resource prices for coal are constant and equal to extraction costs in both scenarios, gross coal prices including a carbon tax increase over time, reducing coal demand in the energy sector.

Regarding the extraction of oil/gas, matters are different because this resource is fully depleted. Over the next five decades, the model estimates an annual extraction of oil/gas equal to 2.4 Gt under laissez-faire.[20] Compared to this, firms mine less crude oil and natural gas over the same time period under optimal policy (2.0 Gt p.a.). In both equilibria, extraction paths exhibit internal resource extraction peaks, which can roughly be labelled as 'peak-oil'. This is in contrast to GHKT who predict monotonically decreasing oil quantities implying that the global economy already exceeded peak-oil. Our model predicts peak-oil not until the year 2055 in the laissez faire equilibrium. This maximum in extracted quantities is even further in the future (year 2075) in the optimal equilibrium. Clearly, however, a precise estimate of a peak-oil point in time strongly depends on parameter choices, model assumptions and initial values and thus need to be interpreted with care.

Moreover, oil production under optimal policy is only initially lower compared to laissez faire, but becomes higher after t = 2115. Intuitively, the tax on emissions reduces oil demand, which results in decreasing oil resource prices over time ensuring complete depletion of oil reserves. This is precisely the forgotten supply side argument advanced by Sinn (2012). In our case, initially curbed oil extraction in the optimal equilibrium is shifted into the future with the total amount extracted unchanged.

## 4.4 Climate stage

The development of total damages as a percentage of global GDP is depicted on the left in Figure 6.[21] Although rather small in the short run, the introduction of a carbon tax has significant effects on damage levels . These effects grow over time and become quite notable in the long run. While damages over the next 50 years are 2.2% of GDP in the laissez faire equilibrium and 1.6% in the optimal allocation, 200 years form now damages are confined to less than 2.3% under optimal taxation, unabated emissions lead to drastic losses and exceed 15% in $t = 2215$.

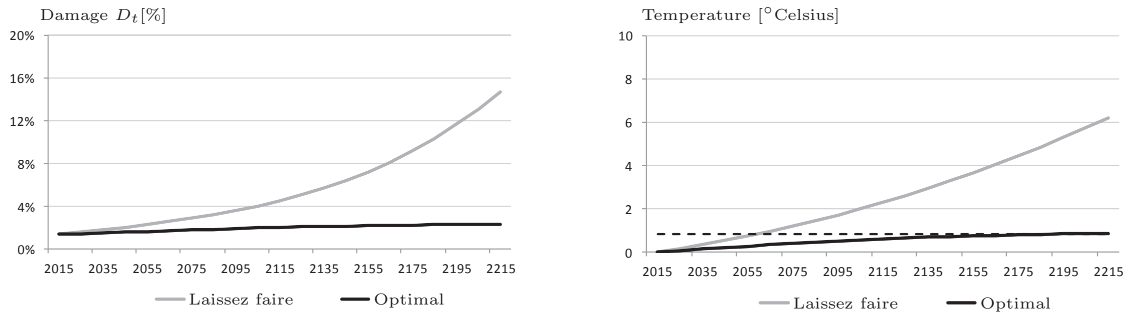
[20]These values imply that our calibrated model underpredicts extracted quantities of oil and gas: According to empirical observations reported by the EIA, annual production of oil and gas was roughly 6.4 Gt in 2014. However, the observed production quantities include oil and especially gas demand for heating by services, commerce and residential. In our model, this direct use of resources for heating purposes is ignored, which leads to a lower total demand of gas.

[21]Using (1) to define $Y_t^{\ell,\mathrm{pot}} := Y_t^\ell/(1-D_t^\ell)$ as potential output in region $\ell$, one observes that $D_t^\ell$ can directly be interpreted as a percentage loss of potential output. Thus, under homogeneous climate damages, $D_t^\ell \equiv D_t$ is the percentage of potential world output $Y_t^{\mathrm{pot}} := \sum_{\ell\in\mathbb{L}} Y_t^{\ell,\mathrm{pot}}$ that is lost due to climate damages.

To study how these damages are accompanied by changes in global mean temperature and thereby assess the effectivity of a carbon tax on fossil fuel with regard to international climate policy aims, we hypothesize a logarithmic relationship between atmospheric $CO_2$ concentration and global mean surface temperature, as proposed in Nordhaus & Yang (1996). For the exact mapping from atmospheric $CO_2$-concentration $S_t$ to global mean surface temperature $TEMP_t$, we follow GHKT by assuming the so-called Arrhenius relation

$$TEMP_t = 3 \log\left(\frac{S_t}{\bar{S}}\right) / \log 2. \tag{41}$$

According to the World Meterological Organization, preliminary data shows that 2016's global temperatures are approximately 1.2 °C above pre-industrial levels, so in order to remain below the postulated 2 °C target, temperature should not rise more than 0.8 °C compared to current levels.




(a) Percentage losses of world GDP

(b) Global temperature relative to 2015

Figure 6: Global temperature and climate damages: Optimal vs. Laissez faire solution.

We portray the predicted change in global temperature and the 2°C target (dashed line) on the right in Figure 6. Under optimal taxation, global temperature increases by 0.6 °C relative to 2015 until 2115. For longer time horizons, the increase continues to be small, reaching 0.86°C in $t = 2215$. These numbers are dramatically different under the laissez faire solution. For this scenario, temperature increases by 2.3°C over the next 100 years (3.97°C compared to pre-industrial levels) and by 6.2°C until $t = 2215$ (7.8°C compared to pre-industrial levels).

Given that temperature in our initial modelling $t = 2015$ is already about 1.2°C higher than the pre-industrial level, the previous findings are in close conformity with the fifth assessment report by the Intergovernmental Panel on Climate Change (2015). This study asserts that the temperature increase relative to the pre-industrial level can still be limited to roughly 2 °C if strict climate policies are adopted while the 'global climate budget' will be exhausted within the next 30 years if no actions are taken. In fact, our model predicts that the two degree target will be reached precisely in $t = 2065$ if emissions are not taxed.

## 4.5 A Pareto-improving transfer policy

All of the previous results only involve the aggregate equilibrium (23) and, therefore, are independent of the transfer policy $\theta = (\theta^\ell)_{\ell \in \mathbb{L}}$ which determines how tax revenue is distributed across regions and which only affects the equilibrium distribution $\mu = (\mu^\ell)_{\ell \in \mathbb{L}}$ of world consumption across regions. In fact, the choice of a particular transfer policy is equivalent to specifying a desired distribution of world consumption across regions. In the following simulation study, we focus on a particular transfer policy which was shown in Hillebrand & Hillebrand (2017) to induce a Pareto improvement relative to the Laissez faire equilibrium if all regions implement the optimal tax. That is, under optimal taxation both regions attain utility higher than at the Laissez faire solution which seems a minimum requirement for regions to agree on a joint climate policy.

Formally, let $\mu^{\mathrm{LF}} = (\mu^{\ell,\mathrm{LF}})_{\ell \in \mathbb{L}}$ denote the consumption shares along the laissez faire equilibrium allocation $\xi^{\mathrm{LF}}$ determined by (16) and $T^{\mathrm{eff}} := \sum_{t=0}^{\infty} q_t \tau_t^{\mathrm{eff}} \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell$ the total discounted tax revenue along the efficient aggregate allocation $\xi^{\mathrm{eff}}$. Define the transfer share of region $\ell$ as

$$\theta^\ell := \frac{\mu^{\ell,\mathrm{LF}} \left[ \sum_{k \in \mathbb{L}} \left( W^k + \Pi^k + r_0 K_0^k \right) + T^{\mathrm{eff}} \right] - W^\ell - \Pi^\ell - r_0 K_0^\ell}{T^{\mathrm{eff}}}, \quad \ell \in \mathbb{L}. \tag{42}$$

Transfers received by region $\ell$ are then determined as $T^\ell := \theta^\ell T^{\mathrm{eff}}$ and induce the target consumption share $\mu^{\ell,\mathrm{LF}}$ from the Laissez faire solution also at the efficient equilibrium. Observe that $\theta^\ell$ may be negative, in which case regions $\ell$ imposes a lump sum tax on domestic consumers to finance transfer payments to the other region. Below we show that this case becomes relevant if climate damages are heterogeneous.

## 4.6 Optimal transfer payments

Several studies suggest that climate damages are more severe in less developed countries, see World Bank (2010). Possible reasons comprise an increased vulnerability due to geographic differences and disadvantages due to a less developed capital stock and inferior knowledge for adaption (Bretschger & Suphaphiphat (2014)).

To incorporate this argument in our study, we add three additional cases to our previous benchmark parametrization to obtain four scenarios. Scenario A corresponds to the previous case with homogeneous climate damages while in the remaining scenarios, climate damages in NOECD countries are slightly (Scenario B), significantly (Scenario C), and dramatically (Scenario D) higher than in OECD countries, respectively. Formally, this is achieved by varying the damage parameters $\gamma^\ell$ in (13). The parameter variations are displayed in the following table together with the resulting consumption shares in the laissez faire equilibrium which will be implemented by the transfer policy. In the benchmark Scenario A, consumption shares essentially coincide with the shares

| | Damage parameters | | | Consumption shares (LF) | |
|---|---|---|---|---|---|
| Scenario | $\gamma_1 \cdot 10^5$ | $\gamma_2 \cdot 10^5$ | Mean | OECD | NOECD |
| A: No differences | 5.3 | 5.3 | 5.3 | 69.90% | 30.20% |
| B: Small differences | 4.4 | 6.2 | 5.3 | 70.37% | 29.63% |
| C: Medium differences | 3.6 | 7.0 | 5.3 | 70.74% | 29.26% |
| D: Large differences | 2.1 | 8.5 | 5.3 | 71.47% | 28.53% |

Table 2: Parameter variations and target consumption shares.

of world GDP produced in each region. As NOECD countries are increasingly exposed to climate damages, however, their share of world consumption gradually reduces by up to 1.6 percentage points in the most extreme Scenario D.

For each scenario, Table 3 displays the tax revenue in each region as a percentage of global revenue together with the transfer shares $\theta^1_{LF}$ and $\theta^2_{LF}$ required to induce the target consumption shares from Table 2. The net transfer displayed in the last column is the difference between tax revenue collected in and transfers received by OECD countries (both expressed as percentages of global tax revenue).

| | Tax revenue | | Transfer shares | | Net Transfer |
|---|---|---|---|---|---|
| Scenario | OECD | NOECD | OECD | NOECD | OECD $\rightarrow$ NOECD |
| A | 65.18% | 34.82% | 28.96% | 71.04% | 36.22% |
| B | 65.01% | 34.99% | 76.30% | 23.70% | -11.29% |
| C | 64.67% | 35.33% | 109.55% | -9.55% | -44.88% |
| D | 64.33% | 35.67% | 182.74% | -82.74% | -118.41% |

Table 3: Tax revenue and optimal transfers between OECD and NOECD countries.

In the benchmark scenario with homogeneous damagevalues, OECD countries collect slightly more than 65% of global tax revenue. To realize the target consumption shares shown in Table 2, they only need about 29% of these revenues resulting in a net transfer of more than 36% to NOECD countries. In the additional three scenarios where NOECD countries are increasingly exposed to climate damages, OECD countries continue to collect about 64% of total tax revenue which declines only slightly as climate damages are more biased towards NOECD countries. As shown in Table 2, however, they are now entitled to receive a higher share of global consumption and, therefore, claim a higher share of tax revenue. Even in Scenario B where differences in climate damage are still moderate, this causes the direction of net transfers to reverse with OECD countries now claiming more than 76% of global tax revenue resulting in a net transfer of 11% from NOECD to OECD countries. This effect continues and becomes even more extreme in Scenarios C and D as climate damages become more severe in NOECD countries. In these case, the transfers needed to realize the target consumption distribution exceed the tax revenue of NOECD countries which must now impose a lump-sum tax (corresponding to a negative transfer) on their domestic consumers to finance these transfer payments. The intuition for this somewhat disturbing result is that climate effects in OECD countries are small if not negligible in this scenario while NOECD countries suffer dramatically. Thus, the latter countries benefit much more from the climate tax and must share his benefit with OECD countries via transfers. The most extreme version of this effect would be where only one poor countries are affected by climate damages and rich countries must be incentivized to take action against climate change via transfers. In essence, NOECD countries must 'pay to survive' under the proposed transfer policy. The general intuition is that countries severely affected by climate damages are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfer payments.

# 5 An Alternative Scenario of Climate Change

## 5.1 Backstop technology

Key assumption in the previous simulation study is the existence of a backstop technology for coal. The first one to introduce such a substitute technology was Nordhaus (1973). Examples of backstop technologies for fossil fuel based energy are for instance solar energy and wind energy. Although widely used and analyzed (Tahvonen & Salo (2001), Tsur & Zemel (2005), Chakravorty, Leach & Moreaux (2012), Valente (2011), Golosov, Hassler, Krusell & Tsyvinski (2014)), this assumption is somewhat controversial, since such a technology has to be developed some time in the future and then enables the world to solve the problem of exhaustibility of resources such as coal.

Relaxing this assumption implies that now coal has a finite resource stock. According to estimates reported by the U.S. Energy Information Administration (EIA) in 2014, global resources of coal (anthracite and lignite) amount to 890 Gt. As coal now also has a scarcity rent, i.e. $v_{2,0} > c_2$, it is optimal to exhaust coal reserves entirely (cf. Lemma 1 in HH). Equation (8) implies positive equilibrium profits from coal extraction. Recall that in the present model, consumers in each region $\ell \in \mathbb{L}$ receive profits of domestic resource sectors. Therefore, the world distribution of coal reserves now becomes relevant, because regional coal extraction determine regional consumers' income. Estimates show that 46% of global coal reserves are located in OECD countries. We thus set $R^1_{0,2} = 414$ and $R^2_{0,2} = 476$ Gt.[22]

[22] The world distribution of coal reserves together with the regional lifetime income levels is relevant for the calculation of optimal transfer payments between regions. Changes in regional income levels affect the regional shares of global consumption under laissez faire, which are our target consumption

Initial resource prices are now such that resource stocks of oil/gas and coal are now both fully exploited. Given our parameter set stated in table 1, this implies prices in the baseline period equal to 356 \$/t (49.8 \$/bbl) for oil and gas and 48.6 \$/t for coal which are close to empirical observations reported by the World Bank (2015a). The rest of the parameter set remains unchanged.We compare the "no backstop laissez faire equilibrium" and the optimal equilibrium in the following.[23]

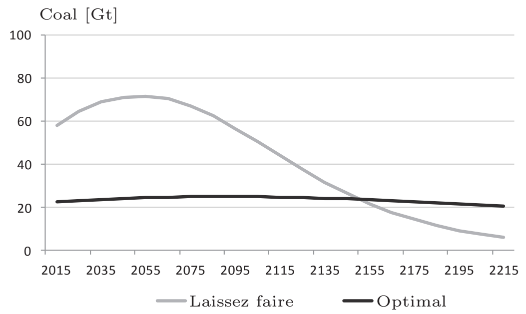
## 5.2 Resource stage

Figure 7 shows the predicted extraction paths of coal (subfigure (a)) and of oil/gas (subfigure (b)) in the laissez faire equilibrium and the optimal equilibrium. We find that under laissez faire, the annual extraction of coal is 5.8 Gt in the baseline period and 2.2 Gt under optimal climate policy. First, this again suggest that the introduction of a tax on fossil fuel in the baseline period leads to reduced extraction initially and to a shift of extracted quantities into the future. Second, the absence of a backstop leads to drastic reduction in coal use: over the next fifity years, accumulated coal extraction amounts to 404 Gt under laissez faire. 200 years from now, 97% (868 Gt) of total coal reserves are used for energy production. This decline in coal use is due to the positive scarcity rent on coal that increases over time according to the Hotelling rule stated in (7) and thereby reduces coal demand. Third, and closely connected to this, we find that extracted quantities of coal without a backstop technology are too low compared to empirical observations reported by U.S. EIA. This supports our suggestion that markets might already expect a backstop technology to be available some time in the future and coal reserves are depleted accordingly.

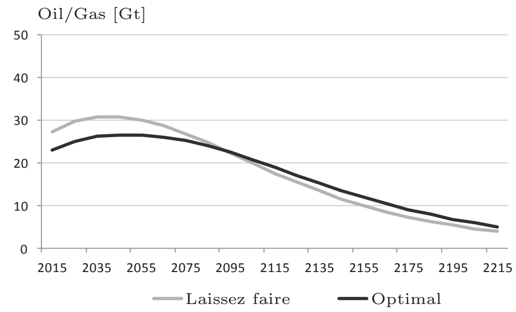
For oil and gas, the model predicts annual extractions equal to 2.7 Gt during the next decade under laissez-faire. Compared to this, firms extract less crude oil and natural gas over the same time period in the optimal policy scenario (2.3 Gt), a carbon tax on fossil fuel however reduces oil/gas use only marginally. Each equilibrium features a 'peak-oil' point in time, a carbon tax shifts this maximum in extracted oil/gas further into the future. Extracted quantities of oil/gas reserves in the optimal equilibrium are only initially lower than in the laissez faire case but becomes higher after $t = 2115$.

shares under optimal policy. However, we find that changes in regional income due to positive profits from coal extraction, lead to negligible changes in regional consumption shares under the laissez-faire. This holds even if coal reserves are completely located in one of the two regions. Thus, our results are robust against arbitrary regional distributions of global coal reserves and we can ignore the analysis of optimal transfer payment in case of no backstop in the following.

[23] The results in the optimal equilibrium are independent of whether we assume the existence of a backstop technology. Given that resource prices evolve according to the classical Hotelling rule (See Hotelling (1931)), it may be optimal not to exhaust resource stocks entirely only if a resource has no scarcity rent, i.e. prices are equal to extraction costs. Since coal reserves are not completely extracted under optimal climate policy, it must be that $v_2 = c_2$, independent of whether a backstop technology is assumed to exist.




(a) Coal extraction




(b) Oil/Gas extraction

Figure 7: Global extraction of exhaustible resources: Optimal vs. Laissez faire solution.

## 5.3 Climate stage

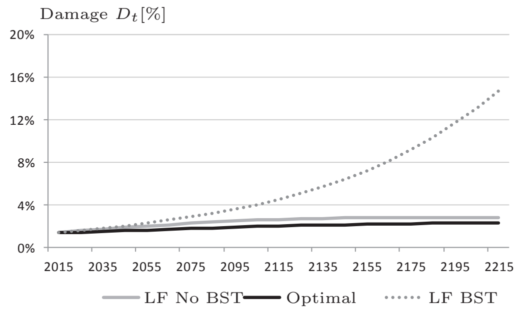
Figure 8 summarizes the corresponding paths of total damages and –using the logarithmic relationship between temperature and atmospheric $CO_2$-concentration from above– global mean surface temperature in the optimal and laissez faire equilibrium. In both equilibria, damages rise over time. Even in the laissez faire equilibrium, however, damages remain below 3% over the next 200 years. Thus a carbon tax can reduce damages only in limited dimensions to a maximum of 2.3% (cf figure 8 (a)).

The dashed horizontal line in figure 8 (b) represents the postulated 2°target (relative to 2015 it is a "0.8°C" target). Under laissez faire, temperature rises to almost 3°C above pre-industrial levels and the 2-degree target is exceeded in 2095. This shows the urgent need to reduce global emissions from burning fossil fuel as soon as possible. Optimal climate policy can help to limit climate change to a rise in temperature of 0.8°C until 2215 compared to 2015 levels. So a carbon tax can limit climate change such that the rise in temperature remains below the 2°target. Altogether, a carbon tax on fossil fuel can help to cut down damages and to abate rise in temperature. The gains are limited, however, compared to a laissez faire outcome.

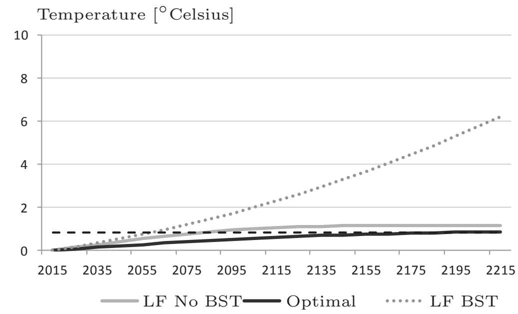
## 5.4 Comparison of results

In both numerical simulations presented in sections 4 and 5 above, differences in fossil fuel use between laissez faire and optimal policy equilibrium are almost entirely driven by differences in coal use, while the pattern of oil/gas use over time changes only marginally.

Now comparing the two laissez faire equilibriums with and without a backstop for coal, we find that time paths of coal extraction crucially depend on this assumption. While coal use accumulated over the next 50 years amounts to 701 Gt if a backstop will appear some time in the future, in fact not later than 2065, corresponding extractions sum up




(a) Percentage losses of world GDP




(b) Global temperature relative to 2015

Figure 8: Global temperature and climate damages: Optimal vs. Laissez faire solution.

to only 400 Gt if no backstop will emerge: Firms in the coal sector produce 7.3 Gt coal annually if a backstop is assumed to appear, compared to 5.7 Gt coal per year in case of no backstop.

The introduction of a backstop implies a constant recource price equal to extraction costs of 43$ per t coal. In contrast, the price for coal increases to 51$/t already in the baseline period if no backstop is allowed to appear in the future. This price effect translates into lower coal use and thus to reduced extractions in the baseline period. Over time, coal price differences increase, leading to diverging extraction paths. With a backstop, coal use increases quickly and more than doubles to 16.3 Gt per year in 50 years, while without a backstop, coal use rises to only 7.0 Gt per year until 2065 and then continually declines due to lower coal demand from the energy sector. 100 years from now, annual extractions of coal amount to 34.5 Gt with backstop compared to only 4.3 Gt without backstop. Interestingly, coal production in case of a backstop is much closer to empirical observations. Assuming the appearance of a backstop then seems to be the empirically more relevant case.

Differences in the dynamics of $CO_2$-emissions and temperature between the two laissez faire equilibriums can be traced back to the corresponding different paths of coal demand. Burning fossil fuel in the two regions lead to aggregate emissions equal to 55 GtC in $t = 0$ if a backstop is allowed to appear, compared to 46 GtC without a backstop. Atmospheric $CO_2$-concentration between the two laissez-faire scenarios differs by only 4.6 GtC (0.5%) in the baseline period. The corresponding initial temperature levels are in both laissez faire equilibriums about 1.6 °C above pre-industrial levels, while temperature in case of a backstop is 0.02 °C higher in comparison to no backstop. The 2°C target is exceeded in both laissez faire equilibriums. If we assume a backstop, climate change exceeds 2°C already in 2055, while if we assume no backstop, the target falls in 2075. This gap in temperature levels increases further over time. 200 years from now the corresponding values are 7.8°C above pre-industrial temperature levels with backstop and only 2.8°C without a backstop.

So although we find that burning fossil fuel, particularly coal, is a threat to climate change, we believe that the quantitative paths generated in section 4 (and in GHKT) tend to overestimate long run damages, since the emergence of a backstop for coal some time in the future is speculative. This is rather surprising, because assuming the emergence of a backstop technology seems to be rather optimistic, while a more conservative view would be to not allow for a backstop technique to appear. This leads to the interesting paradox that a more pessimistic view of technological possibilities in the future generates a more optimistic scenario of future damages caused by climate change.

Summarizing, the quantitative paths generated above hinge crucially on the assumption of a backstop in the coal sector. Qualitatively, this specific technological assumption for coal is less relevant, since even without a backstop, climate change reaches the postulated 2 °target in less than 40 years. This together calls for immediate climate policy action.

# 6 Conclusions

The present paper formulates a dynamic general equilibrium integrated assessment model with two world regions. Each region inhabits an infinitely lived representative consumer. Domestic energy production based on exhaustible resources emits carbon dioxide into the atmosphere. Emssions lead to economic damages through climate change. The optimal policy consists of a Pigouvian tax on carbon emissions and a transfer policy which distributes tax revenue across countries.

We conduct a numerical case study which compares the consequences of climate change for developed and developing countries. We approximate developed countries by OECD member states and developing countries by Non-OECD countries. Thereby, we investigate the effects of the availability of a backstop technology for coal on the paths for resource extraction, emissions and damages due to climate change.

If we allow for such a backstop, temperature exceeds the two-degree target within the next forty years and increases exponentially thereafter under laissez-faire. 200 years from now, output losses due to climate change exceed 15% of global GDP. This results from excessive use of coal making it the major threat to economic growth. If instead no backstop is allowed, coal use is much lower and climate change damages remain below 3% of GDP over the next 200 years. Climate change is therefore still significant but much smaller, stressing the crucial role of this assumption in existing climate studies. So paradoxically, a more optimistic view of technological possibilities in the future leads to a much more pessimistic prospect for the climate problem.

Compared to this, if both regions follow the optimal policy, climate change can be limited to two degrees and OECD and Non-OECD countries can sustain a positive growth rate in the long run, since climate change damages in percent of GDP remain below 2.4%

over the next 200 years. These predictions under optimal policy are independent of the emergence of a backstop technology for coal. Thus our findings are robust against the specific technological assumptions in the coal sector and suggest immediate climate policy action.

The cost of climate policy can be shared by different regions via transfers. We propose a simple transfer policy that takes global income inequalities as given and solves the free-riding behaviour of developed countries by emitting more and more carbon dioxide emissions at the expense of developing countries. The optimal transfer payments under a Pareto-improving transfer policy are chosen such that OECD and Non-OECD countries attain the same consumption share as in the laissez-faire solution. We reveal that this policy may cause net transfers to flow from countries which are severely affected by climate change to countries where damages are less severe: While homogeneous climate damages imply moderate transfer payments from OECD to Non-OECD countries, these payments are reversed as climate damages are increasingly biased towards Non-OECD countries. The intuition for this controversial result is that countries who face greater damages from climate change are in desperate need of climate policies implemented by regions that face only small damages from climate change and, therefore, have little bargaining power in the political process determining transfer payments.

The present work can be extended in different directions. First, it seems fruitful to analyze optimal climate policies in a more elaborate game-theoretic analysis to gain further insights of a regions' incentive to deviate from the optimal policy. Second, the model could be extended to allow for endogenous technical change. Third, another extension is to replace the deterministic setup by a stochastic environment.

# A Computational Details

In this section we explain the details of our algorithm to solve the $M$-dimensional problem $\Phi(\xi^1, \theta) = 0$ defined in Section 2.2 for some vector $\xi^1 \in \Xi^1$ given a fixed vector $\theta = (\mathbf{N}^s, \mathbf{Q}, \mathbf{v}_{-1}, \mathbf{S}_{-1}, \bar{C}_{-1}, \bar{K}) \in \Theta$ of pre-determined variables- For convenience, the time index $t$ is suppressed in this section.

## A.1 The general idea

Our algorithm is based on a very simple idea which is illustrated here for $M = 3$. Suppose we want to determine three real numbers $(x_0, y_0, z_0) \in \mathbb{X} \times \mathbb{Y} \times \mathbb{Z}$ such that $\Phi_1(x_0, y_0, z_0) = \Phi_2(x_0, y_0, z_0) = \Phi_3(x_0, y_0, z_0) = 0$. Then, we can break up this problem into three nested subproblems, referred to as stages, where each stage uses the results from the previous ones.

*Stage I:* Given arbitrary numbers $\hat{y} \in \mathbb{Y}$ and $\hat{z} \in \mathbb{Z}$, consider the problem of determining $\hat{x} \in \mathbb{X}$ such that $\Phi_1(\hat{x}, \hat{y}, \hat{z}) = 0$. If this problem admits a unique solution for any $\hat{y} \in \mathbb{Y}$ and $\hat{z} \in \mathbb{Z}$, we can define a function $\phi_x : \mathbb{Y} \times \mathbb{Z} \longrightarrow \mathbb{X}$ which determines $\hat{x} = \phi_x(\hat{y}, \hat{z})$ such that $\Phi_1(\phi_x(\hat{y}, \hat{z}), \hat{y}, \hat{z}) = 0$ for any $(\hat{y}, \hat{z}) \in \mathbb{Y} \times \mathbb{Z}$.

*Stage II:* Given some value $\tilde{z} \in \mathbb{Z}$, consider the problem of choosing $\tilde{y} \in \mathbb{Y}$ such that $\Phi_2(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ where $\tilde{x} = \phi_x(\tilde{y}, \tilde{z})$ is determined by the previous stage. In other words, given $\tilde{z} \in \mathbb{Z}$ we determine a unique $\tilde{y} \in \mathbb{Y}$ such that $\Phi_2(\phi_x(\tilde{y}, \tilde{z}), \tilde{y}, \tilde{z}) = 0$. If this is again possible for any $\tilde{z} \in \mathbb{Z}$, we can define the solution as a function $\phi_y : \mathbb{Z} \longrightarrow \mathbb{Y}$ such that $\tilde{y} = \phi_y(\tilde{z})$.

*Stage III:* Consider the problem of choosing $\check{z} \in \mathbb{Z}$ such that $\Phi_3(\check{x}, \check{y}, \check{z}) = 0$ where $\check{y} = \phi_y(\check{z})$ and $\check{x} = \phi_x(\check{y}, \check{z})$ are again determined by the functions derived on the previous two stages. If such a solution exists and is unique, setting $z_0 = \check{z}$, $y_0 = \phi_y(z_0)$, and $x_0 = \phi_x(y_0, z_0)$ is the unique solution to the original problem.

## A.2 The algorithm

Our solution approach corresponds exactly to the three-stage structure motivated in the previous example except that the solution sets $\mathbb{X}$, $\mathbb{Y}$, $\mathbb{Z}$ and the functions $\Phi_1$, $\Phi_2$, and $\Phi_3$ are higher-dimensional.

*Stage I*: Given an arbitrary capital allocation $\hat{\mathbf{K}} = (\hat{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{K} := \mathbb{R}_{++}^{L(I+1)}$ and some resource allocation $\hat{\mathbf{X}} = (\hat{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X} := \mathbb{R}_{++}^{LI_x}$, consider the problem of determining a labor allocation[24] $\hat{\mathbf{N}} = (\hat{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{M} := \mathbb{R}_{++}^{L(I+1)}$ which solves the conditions defined by equations (4b), (6b), (29b), and (19) which (after eliminating the wage $w^\ell$ and energy prices $p_i^\ell$) can be stated for all $\ell \in \mathbb{L}$ as

$$
\begin{aligned}
\partial_N F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) &= Q_i^\ell \partial_N F_i(\hat{N}_i^\ell, \hat{K}_i^\ell, \hat{X}_i^\ell) \partial_{E_i} F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) \ \forall i \in \mathbb{I}_x && \text{(A.1a)}\\
&= Q_i^\ell \partial_N F_i(\hat{N}_i^\ell, \hat{K}_i^\ell) \partial_{E_i} F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) \ \forall i \in \mathbb{I}\backslash\mathbb{I}_x && \text{(A.1b)}\\
\sum_{i\in\mathbb{I}} N_i^\ell &= \bar{N}^\ell. && \text{(A.1c)}
\end{aligned}
$$

Energy inputs $(\hat{E}_i^\ell)_{i\in\mathbb{I}}$ in (A.1) are determined from $\hat{\mathbf{N}}$, $\hat{\mathbf{K}}$, and $\hat{\mathbf{X}}$ by (3) and (5) for all $\ell \in \mathbb{L}$. Note that climate damage $1 - D^\ell(\hat{\mathbf{X}}, \mathbf{S}_{-1})$ and final sector productivity $Q_0^\ell$ enter as multiplicative terms in all conditions in (A.1a) and (A.1b) and, therefore, cancel out.

The system (A.1) involves $I + 1$ equations for each region $\ell \in \mathbb{L}$. Define the function $\Phi_1 : \mathbb{M} \times \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{R}^{L(I+1)}$ such that, given $\hat{\mathbf{K}}$ and $\hat{\mathbf{X}}$ , $\hat{\mathbf{N}}$ solves (A.1) if and only if $\Phi_1(\hat{\mathbf{N}}, \hat{\mathbf{K}}, \hat{\mathbf{X}}) = \mathbf{0}$. If such a solution exists and is unique for any $(\hat{\mathbf{K}}, \hat{\mathbf{X}}) \in \mathbb{K} \times \mathbb{X}$, we can define a function $\phi_N : \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{M}$ which determines the solution $\hat{\mathbf{N}} = \phi_N(\hat{\mathbf{K}}, \hat{\mathbf{X}})$.

[24]We define the set of feasible labor allocations as $\mathbb{M}$ because $\mathbb{N}$ is usually reserved for the natural numbers.

To actually compute $\hat{\mathbf{N}}$ in our simulations, we exploit that (A.1) can be solved separately for each region and adopt the following algorithm to compute the solution $\hat{\mathbf{N}}^\ell$ for region $\ell \in \mathbb{L}$. Given a current candidate solution $\tilde{\mathbf{N}}^\ell$ satisfying (A.1c), we compute the associated marginal products of labor $\tilde{w}_i^\ell$ defined by the terms in (A.1a) and (A.1b) in each sector $i \in \mathbb{I}_0$. We then determine the sector $i_{\max}$ with the highest and $i_{\min}$ with the lowest 'wage' and record the current amount of labor allocated to sector $i_{\max}$ as a lower bound and the amount allocated to sector $i_{\min}$ as an upper bound for the actual solution $\hat{N}_i^\ell$ in these sectors. Further, we adjust $\tilde{\mathbf{N}}^\ell$ by shifting a certain amount of labor (which depends on the bounds computed) from the lowest to the highest paying sector. This produces a new candidate solution, for which the process is repeated. Convergence is obtained if all sectors pay the same wage. In our simulations, this approach proved to be a reliable way of solving (A.1).

*Stage II*: Given an arbitrary resource allocation $\tilde{\mathbf{X}} = (\tilde{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X}$, consider the problem of determining a capital allocation $\tilde{\mathbf{K}} = (\tilde{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{K}$ which solves the conditions defined by equations (4a), (6a), (29), and (20) which (after eliminating the capital return $r$ and energy prices $p_i^\ell$) can be stated as

$$(1 - D^\ell(\tilde{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^\ell \partial_K F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) = (1 - D^k(\tilde{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^k \partial_K F_0(\tilde{K}_0^k, \tilde{N}_0^k, (\tilde{E}_i^k)_{i\in\mathbb{I}}) \tag{A.2}$$

for all $\ell, k \in \mathbb{L}$, $\ell \neq k$ and

$$\begin{aligned}
\partial_K F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) &= Q_i^\ell \partial_K F_i(\tilde{K}_i^\ell, \tilde{N}_i^\ell, \tilde{X}_i^\ell)\partial_{E_i} F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) \;\forall i \in \mathbb{I}_x && \text{(A.3a)}\\
&= Q_i^\ell \partial_K F_i(\tilde{K}_i^\ell, \tilde{N}_i^\ell)\partial_{E_i} F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) \;\forall i \in \mathbb{I}\backslash\mathbb{I}_x && \text{(A.3b)}\\
\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_0} K_i^\ell &= \bar{K} && \text{(A.3c)}
\end{aligned}$$

for all $\ell \in \mathbb{L}$. Here, $\tilde{\mathbf{N}} = (\tilde{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_N(\tilde{\mathbf{K}}, \tilde{\mathbf{X}})$ is determined by Stage I and energy inputs $(\tilde{E}_i^\ell)_{i\in\mathbb{I}}$ in both (A.2) and (A.3) follow from (3) and (5) for all $\ell \in \mathbb{L}$. Equation (A.2) equates marginal products of capital in final production across all regions while (A.3) equates marginal products of capital across all sectors within each region. Both conditions together thus equalize capital returns across all sectors and regions.

The system defined by (A.2) and (A.3) consist of $L - 1 + LI + 1 = L(I + 1)$ equations. Define the function $\Phi_2 : \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{R}^{L(I+1)}$ such that, given $\tilde{\mathbf{X}}$, $\tilde{\mathbf{K}}$ solves (A.2) and (A.3) if and only if $\Phi_2(\tilde{\mathbf{N}}, \tilde{\mathbf{K}}, \tilde{\mathbf{X}}) = \mathbf{0}$ where $\tilde{\mathbf{N}} = \phi_N(\tilde{\mathbf{K}}, \tilde{\mathbf{X}})$. If such a solution exists and is unique for any $\tilde{\mathbf{X}} \in \mathbb{X}$, there exists a function $\phi_K : \mathbb{X} \longrightarrow \mathbb{K}$ which determines this solution as $\tilde{\mathbf{K}} = \phi_K(\tilde{\mathbf{X}})$, i.e., $\Phi_2(\phi_N(\phi_K(\tilde{\mathbf{X}}), \tilde{\mathbf{X}}), \phi_K(\tilde{\mathbf{X}}), \tilde{\mathbf{X}}) = \mathbf{0}$ for all $\tilde{\mathbf{X}} \in \mathbb{X}$.

To compute $\tilde{\mathbf{K}}$, it is tempting to follow a similar strategy as for Stage I, defining for any candidate solution $\check{\mathbf{K}}$ the marginal capital products $\check{r}_i^\ell$ in each region and sector and shifting capital from the lowest to the highest paying sector. Due to (A.3c), this problem can no longer be solved separately for each region but requires shifting capital around globally across all regions and sectors. Unfortunately, this approach caused some potential instability for our algorithm which, occasionally, did not converge to the desired solution. To remedy this problem, we therefore break up Stage II into two steps.

In the first step, we fix a capital distribution $(\bar{K}^\ell)_{\ell\in\mathbb{L}} \in \mathbb{R}^L_{++}$ across regions where $\bar{K}^\ell > 0$ is total capital employed in region $\ell$ and $\sum_{\ell\in\mathbb{L}} \bar{K}^\ell = \bar{K}$. We then solve (A.3) separately for each region $\ell$, replacing (A.3c) by the condition

$$\sum_{i\in\mathbb{I}_0} K_i^\ell = \bar{K}^\ell. \tag{A.4}$$

This step allows us to essentially employ the same routine as on Stage I and determine for each region $\ell \in \mathbb{L}$ a capital allocation $\check{\mathbf{K}}^\ell = (\check{K}_i^\ell)_{i\in\mathbb{I}_0}$ which induces a uniform capital return $\check{r}^\ell$ across sectors.

In the second step, we adjust the capital distribution $(\bar{K}^\ell)_{\ell\in\mathbb{L}}$ based on the regional capital returns $\check{r}^\ell$ computed in Step 1 shifting capital from the lowest to the highest paying region and then repeating the computation of Step 1. The desired solution is reached if equation (A.2) is satisfied and capital returns are identical for all regions.

It turned out that this two-step strategy completely eliminates the previous convergence problems.

*Stage III*: At the final stage, we determine the resource allocation $\check{\mathbf{X}} = (\check{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X}$ such that equations (4c), (7), and (26) are satisfied. After eliminating energy prices using (2c), we obtain the following condition which must hold for all $\ell \in \mathbb{L}$ and $i \in \mathbb{I}_x$:

$$(1 - D^\ell(\check{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^\ell \partial_{E_i} F_0(\check{K}_0^\ell, \check{N}_0^\ell, (\check{E}_i^\ell)_{i\in\mathbb{I}}) Q_i^\ell \partial_X F_i\big(\check{K}_i^\ell, \check{N}_i^\ell, \check{X}_i^\ell\big) = c_i + \check{r}(v_{i,-1} - c_i) + \zeta_i \check{\tau} \tag{A.5}$$

where the factor allocation $\check{\mathbf{K}} = (\check{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_K(\check{\mathbf{X}})$ and $\check{\mathbf{N}} = (\check{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_N(\check{\mathbf{K}}, \check{\mathbf{X}})$ is determined from $\check{\mathbf{X}}$ by Stages I and II. The factor allocation determines energy inputs $(\check{E}_i^\ell)_{i\in\mathbb{I}}$ in (A.5) by (3) and (5), the tax rate $\check{\tau}$ by using (28) in (26), and the global capital return $\check{r}$ as the marginal product of capital in any region or sector.

Noting that the r.h.s. in (A.5) is independent of $\ell$, the system (A.5) involves $LI_x$ equations that can potentially be solved to determine a unique solution $\check{\mathbf{X}}$. Our solution strategy is to determine for any candidate solution $\acute{\mathbf{X}}$ the induced factor allocation $\acute{\mathbf{K}}$ and $\acute{\mathbf{N}}$ and the r.h.s. in (A.5) as $\acute{\pi}_i := c_i + \acute{r}(v_{i,-1} - c_i) + \zeta_i \acute{\tau}$ for each resource $i \in \mathbb{I}_x$. Then, for each $\ell \in \mathbb{L}$, we adjust $X_i^\ell$ based on the discrepancy between $\acute{\pi}_i$ and the marginal product on the l.h.s. in (A.5). This produces a new candidate solution for which the previous computations can be repeated until convergence obtains and the solution $\check{\mathbf{X}}$ is reached. ■

# References


Berlemann, M. & J. Wesselhoeft (2014): "Estimating Aggregate Capital Stocks Using the Perpetual Inventory Method -A Survey of Previous Implementations and

New Empirical Evidence for 103 Countries", *Review of Economics*, 65, 1–34.

Bretschger, L. & N. Suphaphiphat (2014): "Effective climate policies in a dynamic North-South model", *European Economic Review*, 69, 59–77.

Chakravorty, U., A. Leach & M. Moreaux (2012): "Cycles in nonrenewable resource prices with pollution and learning-by-doing", *Journal of Economic Dynamics and Control*, 36, 1448–1461.

Department of Energy & Climate Change (2013): *Electricity Generation costs 2013*. Department of Energy & Climate Change, United Kingdom, London.

Earth System Research Laboratory (2016): "Trends in atmospheric Carbon Dioxide", https://www.esrl.noaa.gov/gmd/ccgg/trends/global.html.

Federal Institute for Geosciences and Natural Resources (2015): "Energy Study 2015. Reserves, Resources and Availability of Energy Resources", http://www.bgr.bund.de/EN/Themen/Energie/Produkte/energy_study_2015_summary_en.html.

Golosov, M., J. Hassler, P. Krusell & A. Tsyvinski (2014): "Optimal Taxes on Fossil Fuel in General Equilibrium", *Econometrica*, 82(1), 41–88.

Hassler, J. & P. Krusell (2012): "Economics And Climate Change: Integrated Assessment In A Multi-Region World", *Journal of the European Economic Association*, 10(5), 974–1000.

Hassler, J., P. Krusell & A. A. Smith (2016): "Chapter 24: Environmental Macroeconomics", in *Handbook of Macroeconomics, 1st Edition*, ed. by J. B. Taylor & H. Uhlig. Elsevier (North Holland Publishing Co.), Amsterdam.

Hillebrand, B. (1997): "Stromerzeugungskosten neu zu errichtender konventioneller Kraftwerke", Discussion Paper 47, Rheinisch-Westfälisches Institut für Wirtschaftsforschung.

Hillebrand, E. & M. Hillebrand (2017): "Optimal Climate Policies in a Dynamic Multi-Country Equilibrium Model", Working paper, Gutenberg University Mainz.

Hotelling, H. (1931): "The Economics of Exhaustible Resources", *Journal of Political Economy*, 39(2), 137–175.

Intergovernmental Panel on Climate Change (2015): "Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change", Core Writing Team, R.K. Pachauri and L.A. Meyer (eds.)]. IPCC, Geneva, Switzerland, 151 pp. Fifth Assessment Report.

International Energy Agency (2010): *World Energy Outlook 2010*. OECD Publishing, Paris.

——— (2014): *Key World Energy Statistics 2014*. OECD Publishing, Paris.

International Monetary Fund (2015): "World Economic Outlook 2015", http://www.imf.org/external/pubs/ft/weo/2015/01/.

Loeschel, A. & V. Otto (2009): "Technological uncertainty and cost effectiveness of CO2 emission reduction", *Energy Economics*, 31, 4–17.

Nordhaus, W. (1973): "The Allocation of Energy Resources", Discussion paper, Brookings Papers on Economic Activity.

——— (1977): "Growth and Climate: The Carbon Dioxide Problem", *American Economic Review*, 67(1), 341–346.

——— (2011): "Integrated Economic and Climate Modeling", Discussion paper, Cowles Foundation Discussion Paper No. 1839.

Nordhaus, W. & Z. Yang (1996): "A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies", *American Economic Review*, 86, 741–765.

Sinn, H.-W. (2012): *The Green Paradox. A Supply-Side Approach to Global Warming.* The MIT Press, Cambridge, Massachusetts, and London, GB.

Stern, D. I. (2012): "Interfuel substitution: A meta-analysis", *Journal of Economic Surveys*, 26, 207–331.

Tahvonen, O. & S. Salo (2001): "Economic growth and transition between renewable and nonrenewable energy resources", *European Economic Review*, 45, 1379–1398.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer & J. d. V. Gaaitzen (2015): "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production", *Review of International Economics*, 23, 575–605.

Tsur, Y. & A. Zemel (2005): "Scarcity, growth and R&D", *Journal of Environmental Economics and Management*, 49, 484–499.

U.S Bureau of Economic Analysis (2007): "Input-Output Accounts Data", http://www.bea.gov/industry/io-annual.htm.

Valente, S. (2011): "Endogenous growth, backstop technology adoption, and optimal jumps", *Macroeconomic Dynamics*, 15, 293–325.

World Bank (2010): "World Development Report", Development and Climate Change, Washington.

——— (2015a): "Global Economic Monitor (GEM) Commodities", http://databank.worldbank.org/data/databases/commodity-price-data.

——— (2015b): "World Development Indicator (WDI)", http://databank.worldbank.org/data/databases/.