A Simulation Study of Global Warming and Optimal Climate Policy*

Elmar Hillebrand† Marten Hillebrand‡

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Abstract

This paper presents a quantitative study of climate policies using a dynamic general equilibrium model with multiple regions. We develop a generally applicable algorithm to compute the equilibrium under alternative climate tax policies. Our simulation study quantifies the economic, environmental, and social effects of introducing an optimal climate policy for OECD and Non-OECD countries. Optimal taxation keeps the increase in temperature below two degrees and permits both regions to sustain positive long-run growth. Laissez faire causes temperature to exceed the two-degree target within the next forty years, leading to massive damages and output losses. This result, however, hinges crucially on the arrival of a backstop technology which provides a perfect substitute for coal within the next fifty years. Otherwise, climate damages are significantly smaller, stressing the crucial role of this assumption in existing studies. Transfer payments which Pareto-improve the laissez faire equilibrium flow from OECD to Non-OECD countries under homogeneous climate damages. This direction reverses if climate damages become increasingly biased towards Non-OECD countries.

JEL classification: E10, E61, H21, H23, Q43, Q54

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†EEFA Research Institute, Windthorststr. 13, 48143 Muenster, Germany, email: e.hillebrand@eefa.de

‡Department of Economics, Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany, email: Hillebrand@em.uni-frankfurt.de (corresponding author)

Introduction

Global warming constitutes one of the biggest economic, environmental, and social challenges of the twenty-first century. To provide economic guidance to the political process and assess the consequences of alternative climate policies, the class of *integrated assessment (IA)* models (see Nordhaus (2011) or Hassler et al. (2016) for a survey) offers a theoretical framework which incorporates the full interactions between the economic system and climate variables. This approach permits to quantify the impact of policies on economic variables such as output, investment, or employment as well as on climate variables such as emissions, carbon concentration, or temperature. The present paper uses the multi-country IA model developed in Hillebrand & Hillebrand (2017) to quantify the economic, environmental, and social consequences of climate change for OECD and NOECD countries and the effects of implementing an optimal climate policy.

Most of the existing IA-models in the literature are based on the DICE framework pioneered by Nordhaus (1977) which was extended to the multi-region RICE model in Nordhaus & Yang (1996). While the DICE/RICE framework offers a very detailed description of the climate system with its various feedback structures and layers, the economic production process abstracts from at least two features we think play a key role in the climate problem. First, the model does not describe the depletion of exhaustible resources and, crucially, does not impose any constraints on the stocks of fossil resources. Thus, the stocks of coal and oil are effectively infinite which allows for potentially unbounded increases in global temperature and climate damage. Second, the DICE/RICE framework does not explicitly include the production of energy goods such as power, heat, or transportation. But it is this stage of the production process at which most emissions occur and where climate policy aims to induce a transition from dirty to clean technologies as, e.g., in Acemoglu et al. (2012). Such a transition and the induced sectoral changes can therefore not be studied in the DICE/RICE framework.

An alternative framework with a more detailed description of the production stage and explicit constraints on the extraction of fossil fuels is developed in Golosov et al. (2014), henceforth GHKT. Their main result determines the optimal (Pigovian) tax on fossil emissions in closed form. They also present a comprehensive simulation study to quantify the effects of implementing this optimal climate policy. The GHKT model treats the world as a single region and, therefore, does not allow to study how global warming and alternative climate policies affect different regions. Moreover, although their specification of the production process distinguishes a finite number of clean and dirty technologies, the outputs of these sectors are constrained by the available stocks of fossil fuels. Thus, it seems more natural to interpret these sectors as resource sectors which *supply* exhaustible resources like oil or coal rather than energy sectors which *demand* such resources to produce outputs like electricity or heat. Hence, their model does not allow to study sectoral changes at the level of energy production in the above sense.

The model developed in Hillebrand & Hillebrand (2017) and henceforth referred to as the HH-model adopts essentially the same modelling philosophy as GHKT but extends their framework along various dimensions. First, like the RICE model, it distinguishes an arbitrary number of regions which differ with respect to their state of economic development, factor endowments, and climate damages. Second, it enhances the description of the production process by distinguishing the extraction of resources and the production of energy in each region. This permits to separate the impact of climate policies on energy production and resource extraction in each region and to impose constraints on resource extraction based on empirically plausible resource stocks. At the same time, the HH-model retains the virtue of analytical tractability which was exploited in Hillebrand & Hillebrand (2017) to derive an optimal climate policy. This policy consists of an optimal emissions tax and a Pareto-improving transfer policy which makes each region better-off relative to the laissez faire solution.

The present paper employs a calibrated parametrization of the HH-model to quantify the effects of implementing the optimal climate policy. The analysis consists of three parts. The first part reviews the underlying model and identifies its forward-recursive structure to develop a numerical algorithm which computes the equilibrium solution for a given climate policy. Our computational method is applicable for a large class of climate policies and an arbitrary number of regions, production sectors, and exhaustible resources. It therefore offers a general methodological contribution that goes beyond its application in the present paper.

The second and third part present the results of our simulation study. We consider the case with two-regions which are calibrated to match the main economic features of OECD and NOECD countries. This choice of regions is motivated by the idea of analyzing how climate policy affects rich and poor countries in potentially different ways as in Bretschger & Suphaphiphat (2014). To infer the economic and environmental consequences of implementing the optimal climate policy, we compare selected economic and climate variables under the optimal climate policy to the laissez faire solution without policy intervention.

The social dimension of climate policy in our study is represented by the transfer payments between regions. Here, we confine attention to the Pareto-improving transfer policy derived in Hillebrand & Hillebrand (2017). While our benchmark scenario assumes homogeneous climate damages across regions, we also compute optimal transfers under alternative scenarios where damages are increasingly biased towards NOECD countries. Including these additional scenarios is based on the notion that poorer countries may be more severely affected by climate damages than developed countries which has wide empirical support, cf. World Bank (2010). In our simulations, we find that homogeneous climate damages imply moderate transfer payments from OECD to Non-OECD countries. However, the direction of transfers reverses as climate damages are increasingly biased towards Non-OECD countries. Thus, regions which suffer *more* damage from climate change receive *less* transfers. The intuition for this surprising (and potentially controversial) result is that countries severely affected by climate damages are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfer payments.

A key assumption made in our benchmark scenario and most quantitative studies of climate change is the existence of a backstop technology for coal. Originally introduced by Nordhaus (1973), such a technology assumes that coal use is not constrained by exhaustibility and, therefore, has no scarcity rent rendering its supply effectively infinite. Although widely used and analyzed in the literature (cf. Chakravorty et al. (2012), Tahvonen & Salo (2001), Tsur & Zemel (2005), Valente (2011), and GHKT), the assumption remains somewhat controversial. Thus, a natural experiment is to analyze how the numerical results change if we drop the assumption of a backstop technology. This constitutes the final third part of our simulation study.

The optimal solution in our benchmark scenario does not exhaust the existing stock of coal and, therefore, is not affected by this modification. For the laissez faire solution, however, we find that the quantitative results change substantially and climate damages are still sizeable but much less severe than in the benchmark case. Thus, we infer that the quantitative results obtained in the benchmark scenario hinge crucially on the hypothesized emergence of a backstop technology. Ironically, a more optimistic view about technological innovation which solves the exhaustibility of resources leads to a much more pessimistic prospect for the climate problem.

Our simulation results contribute to the existing literature in the following ways. First, we offer a calibrated simulation study for OECD and NOECD countries which quantifies how introducing an optimal emissions tax affects production output and the growth path in each region. Second, we provide a detailed analysis of how climate policy affects the composition of energy production based on clean and dirty technologies and quantify the induced sectoral changes in each region. Third, we analyze how the optimal tax changes the global extraction of oil/natural gas and coal and climate variables such as emissions and temperature. Fourth, we analyze how the results depend on the emergence of a backstop technology and uncover the key role this assumption plays to obtain the quantitative results in our and most existing studies in the literature. Finally, we quantify the direction and size of optimal transfers between OECD and NOECD countries under the optimal climate policy.

The paper is organized as follows. Section 1 introduces the model. Section 2 develops an algorithm to solve for the equilibrium under alternative specifications of a climate tax policy. Section 3 describes our calibration strategy of the model's parameters. The numerical results are presented in Sections 4 and 5 where we distinguish two scenarios depending on whether coal will be replaced by a backstop technology or not. Section 6 concludes, computational details are relegated to Appendix A.

1 The Model

1.1 Regions and sectors

The world economy evolves in discrete time $t \in \{0, 1, 2, \ldots\}$ and is divided into $L \geq 2$ regions indexed by $\ell \in \mathbb{L} := \{1, \ldots, L\}$. Each region $\ell \in \mathbb{L}$ pursues its own interests and takes autonomous political decisions. Regions are geographically or institutionally separated, which imposes certain restrictions on trade between them.

The production process in each region $\ell \in \mathbb{L}$ decomposes into three stages. The first stage is the *final sector* which produces a consumable output commodity using labor, capital, and energy goods and services. The second stage consists of *energy sectors* which produce these goods and services using labor, capital, and exhaustible resources. The third stage consists of *resource sectors* which extract the domestic stock of exhaustible resources. The production side is complemented by a climate model and a description of the consumption sector in each region.

The following sections introduce these building blocks formally and derive the decentralized equilibrium solution for a given climate policy.[1]

1.2 Production sectors

Sectoral structure
Production sectors in region $\ell \in \mathbb{L}$ are identified by the index $i \in \mathbb{I}_0 := \{0, 1, \ldots, I\}$. Sector $i = 0$ is the *final sector* which produces a consumable output good in each period that can also be invested to become capital in the following period. Production sectors $i \in \mathbb{I} := \mathbb{I}_0 \backslash \{0\}$ are *energy sectors* which supply energy goods like electricity and heat or services like fuel-based transportation as inputs to final good production. We further denote by $\mathbb{I}_x \subset \mathbb{I}$ the set of energy sectors which base their production on exhaustible resource like coal, oil, and natural gas. As burning exhaustible resources in energy production causes emissions, sectors $\mathbb{I}_x$ are the sectors responsible for climate change. Production in the other energy sectors is based on renewable sources like wind, water, and solar energy which do not enter as production inputs and do not cause emissions.

Each sector $i \in \mathbb{I}_0$ consist of a single representative firm which employs labor $N^\ell_{i,t} \geq 0$ and capital $K^\ell_{i,t} \geq 0$ as production factors in period t. The amount of exhaustible resources used by sector $i \in \mathbb{I}_x$ is denoted $X^\ell_{i,t} \geq 0$ and is an essential input to production. All production technologies are based on time-invariant production functions F_i which are linear homogeneous, twice continuously differentiable, strictly increasing, and concave.

[1]See Hillebrand & Hillebrand (2017) for additional details on the model and the results presented in this section.

Productivity in sector $i \in \mathbb{I}_0$ in region $\ell \in \mathbb{L}$ is denoted $Q^\ell_{i,t} > 0$ and may be time- and country-specific. Denote by $w^\ell_t > 0$ the wage and $p^\ell_{i,t} > 0$ the price of energy type $i \in \mathbb{I}$ in period t. As labor and energy outputs will be immobile across regions, their prices will, in general, be region-specific. By contrast, capital and exhaustible resources are traded on international markets implying that their prices are not region-specific. The (rental) price of capital at time $t \geq 0$ is denoted $r_t > 0$ and the world price of the exhaustible resource used by sector $i \in \mathbb{I}_x$ as $v_{i,t} > 0$. Conceptually, all transactions take place in $t = 0$ and all prices in period t are denominated in units of time t consumption.

Final sector

Sector $i = 0$ in region $\ell \in \mathbb{L}$ uses labor, capital, and energy goods and services $(E^\ell_{i,t})_{i \in \mathbb{I}}$ to produce output Y^ℓ_t in period $t \geq 0$ according to the production technology

$$Y^\ell_t = (1 - D^\ell_t) Q^\ell_{0,t} F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i \in \mathbb{I}}). \tag{1}$$

Here, $D^\ell_t \in [0, 1[$ is an index of climate damage which will be a function of total CO_2-concentration in the atmosphere specified below. Given these parameters and prices for labor, capital, and energy inputs, the final sector solves the following atemporal decision problem in each period $t \geq 0$:

$$\max_{(K,N,E_1,\ldots,E_I) \in \mathbb{R}^{2+I}_+} \left\{ (1 - D^\ell_t) Q^\ell_{0,t} F_0(K, N, (E_i)_{i \in \mathbb{I}}) - w^\ell_t N - r_t K - \sum_{i \in \mathbb{I}} p^\ell_{i,t} E_i \right\}.$$

The profit maximizing solution in period $t \geq 0$ is characterized by the standard first order conditions:

$$(1 - D^\ell_t) Q^\ell_{0,t} \partial_K F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i \in \mathbb{I}}) = r_t \tag{2a}$$

$$(1 - D^\ell_t) Q^\ell_{0,t} \partial_N F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i \in \mathbb{I}}) = w^\ell_t \tag{2b}$$

$$(1 - D^\ell_t) Q^\ell_{0,t} \partial_{E_i} F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i \in \mathbb{I}}) = p^\ell_{i,t} \quad \forall i \in \mathbb{I}. \tag{2c}$$

Exhaustible energy sectors

Each sector $i \in \mathbb{I}_x$ is uniquely identified by the underlying resource on which production is based (like 'coal' used for 'coal-fired power generation' or 'oil' used to provide 'fuel-based transportation services'). The technology used by sector $i \in \mathbb{I}_x$ takes the form

$$E^\ell_{i,t} = Q^\ell_{i,t} F_i(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}). \tag{3}$$

Burning exhaustible resources generates emissions proportional to their usage in production. Energy sectors thus represent the production stage at which emissions are potentially generated. The amount of emissions generated by using $X^\ell_{i,t} \geq 0$ in production are $Z^\ell_{i,t} = \zeta_i X^\ell_{i,t}$ where ζ_i is the specific carbon-content of resource i. To combat climate damages, all regions impose a uniform climate tax $\tau_t \geq 0$ to be paid by energy sectors per unit of CO_2 emitted in period t. Firms in these sectors take this tax together with productivity and prices relevant to their decision as given parameters. Their decision problem solved in period $t \geq 0$ reads:

$$\max_{(K,N,X)\in\mathbb{R}^3_+} \Big\{ p^\ell_{i,t} Q^\ell_{i,t} F_i(K,N,X) - w^\ell_t N - r_t K - (v_{i,t} + \tau_t \zeta_i) X \Big\}.$$

Clearly, the profit maximizing solution becomes independent of τ_t if $\zeta_i = 0$, i.e., the firm employs a clean technology. The first order conditions necessary and sufficient for an optimal solution are given by:

$$p^\ell_{i,t} Q^\ell_{i,t} \partial_K F_i\big(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}\big) = r_t \tag{4a}$$
$$p^\ell_{i,t} Q^\ell_{i,t} \partial_N F_i\big(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}\big) = w^\ell_t \tag{4b}$$
$$p^\ell_{i,t} Q^\ell_{i,t} \partial_X F_i\big(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t}\big) = v_{i,t} + \zeta_i \tau_t. \tag{4c}$$

Renewable energy sectors
Production of firms $i \in \mathbb{I}\backslash\mathbb{I}_x$ are based on *renewable sources* like wind, solar energy, etc. which do not enter as inputs to production. Their technology is given by

$$E^\ell_{i,t} = Q^\ell_{i,t} F_i(K^\ell_{i,t}, N^\ell_{i,t}) \tag{5}$$

Each firm $i \in \mathbb{I}\backslash\mathbb{I}_x$ in the renewable energy sector takes productivity and the relevant prices as given to solve the following profit maximization problem in period $t \geq 0$:

$$\max_{(K,N)\in\mathbb{R}^2_+} \Big\{ p^\ell_{i,t} Q^\ell_{i,t} F_i(K,N) - w^\ell_t N - r_t K \Big\}.$$

The first order conditions for profit maximization in period $t \geq 0$ are given by

$$p^\ell_{i,t} Q^\ell_{i,t} \partial_K F_i\big(K^\ell_{i,t}, N^\ell_{i,t}\big) = r_t \tag{6a}$$
$$p^\ell_{i,t} Q^\ell_{i,t} \partial_N F_i\big(K^\ell_{i,t}, N^\ell_{i,t}\big) = w^\ell_t. \tag{6b}$$

Resource sectors
Exhaustible resources are uniquely identified by the energy sector $i \in \mathbb{I}_x$ which uses this resource in production. In each region $\ell \in \mathbb{L}$, there exists a single firm which extracts resources of type $i \in \mathbb{I}_x$. In period $t \geq 0$, this firm extracts resources $X^{\ell,s}_{i,t} \geq 0$ (to be distinguished from the amount $X^\ell_{i,t}$ demanded by energy sector $i \in \mathbb{I}_x$ in that region) and sells them in the global resource market at the price $v_{i,t}$. Firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R^\ell_{i,0} \geq 0$ together with the selling prices $(v_{i,t})_{t\geq 0}$ as a given parameter. Their objective is to maximize the discounted stream of future profits. As the economy is deterministic, profits in period $t \geq 0$ are discounted to period zero by the discount factor $q_t := \prod_{s=1}^t r_s^{-1}$ where $q_0 = 1$. With this notation, the decision problem solved by resource sector $i \in \mathbb{I}_x$ reads

$$\max_{(X^{\ell,s}_{i,t})_{t\geq 0}} \left\{ \sum_{t=0}^\infty q_t (v_{i,t} - c_i) X^{\ell,s}_{i,t} \;\middle|\; \sum_{t=0}^\infty X^{\ell,s}_{i,t} \leq R^\ell_{i,0},\ X^{\ell,s}_{i,t} \geq 0\ \forall t \geq 0 \right\}.$$

If $R^\ell_{i,0} > 0$, the linearity of the extraction technology implies that an interior optimal extraction plan exists if and only if resource prices satisfy $v_{i,0} \geq 0$ and the Hotelling rule

$$v_{i,t} = c_i + r_t(v_{i,t-1} - c_i) \quad \forall t > 0. \tag{7}$$

Clearly, only if $v_{i,0} = c_i$ may it be optimal not to exhaust the entire stock of resources. In either case, (7) permits equilibrium profits of resource sector $i \in \mathbb{I}_x$ to be written as

$$\Pi^\ell_i = (v_{i,0} - c_i)R^\ell_{i,0}. \tag{8}$$

Climate policy

A climate policy determines the sequence of emissions taxes $\tau = (\tau_t)_{t\geq 0}$ which all regions are assumed to impose. The revenue from taxing emissions is then distributed as lump-sum transfers to consumers in each region. We assume that regions agree on a time-invariant transfer policy $\theta = (\theta^\ell)_{\ell\in\mathbb{L}}$ satisfying $\sum_{\ell\in\mathbb{L}} \theta^\ell = 1$ which determines the share θ^ℓ of tax revenue received by region ℓ. This transfer policy constitutes the second part of a climate policy. Note that the case $\theta^\ell < 0$ is not excluded in this definition, in which case consumers in region ℓ are taxed to finance transfers received by other countries. Thus, the previous specification also allows for international redistribution via lump-sum taxation. It follows that the total discounted transfers received by consumers in region ℓ can be expressed as

$$T^\ell = \theta^\ell \sum_{t=0}^{\infty} q_t \tau_t \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X^\ell_{i,t}. \tag{9}$$

1.3 Climate model

Emissions of CO_2 are generated by using ('burning') exhaustible fossil fuels like coal, oil, and gas in the production of energy. The amount of CO_2 generated by using one unit of exhaustible resource $i \in \mathbb{I}_x$ is physically determined by its carbon-content $\zeta_i \geq 0$. In particular, $\zeta_i = 0$ if the resource does no generate emissions, like uranium in the case of nuclear energy production. Total emissions in period t measured in units of CO_2 are given by

$$Z_t := \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X^\ell_{i,t}. \tag{10}$$

Adopting the specification from GHKT, the climate state in period t consists of permanent and non-permanent CO_2 in the atmosphere and is denoted by $\mathbf{S}_t = (S_{1,t}, S_{2,t})$. Given an emissions sequence $\{Z_t\}_{t\geq 0}$ determined by (10), the climate state evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \tag{11a}$$
$$S_{2,t} = (1-\phi)S_{2,t-1} + (1-\phi_L)\phi_0 Z_t \tag{11b}$$

Specification (11) assumes that a share $0 \leq \phi_L < 1$ of emissions become permanent CO_2. Out of the remaining emissions, a share ϕ_0 becomes non-permanent CO_2 which decays at constant rate $0 < \phi < 1$ while the remaining share $1 - \phi_0$ leaves the atmosphere (see GHKT for details). Total concentration of CO_2 at time t is given by

$$S_t = S_{1,t} + S_{2,t}. \tag{12}$$

Denote by $\bar{S} > 0$ the pre-industrial level of CO_2 in the atmosphere. Climate damage in region ℓ is determined by total concentration of CO_2 in the atmosphere according to the function

$$D_t^\ell = D^\ell(S_t) := 1 - \exp\{-\gamma^\ell(S_t - \bar{S})\}, \quad \gamma^\ell > 0 \tag{13}$$

which corresponds to the choice in GHKT.[2] Reginal differences in climate damage thus enter via region specific parameters γ^ℓ, $\ell \in \mathbb{L}$.

1.4 Consumption sector

The consumption sector in each region $\ell \in \mathbb{L}$ consists of a single representative household which supplies labor and capital to the production process and decides about consumption and capital formation taking factor prices as given. In addition, the consumer is entitled to receive all profits from domestic firms and transfers from the government. A direct consequence of the linear-homogeneity of the production functions F_i is that profits in final production and all energy sectors are zero. Thus, by (8) the total discounted profit income of the household in region $\ell \in \mathbb{L}$ is

$$\Pi^\ell = \sum_{i \in \mathbb{I}_x} \Pi_i^\ell = \sum_{i \in \mathbb{I}_x} (v_{i,0} - c_i) R_{i,0}^\ell. \tag{14}$$

The household's preferences over non-negative consumption sequences $(C_t^\ell)_{t \geq 0}$ are represented by a standard time-additive utility function

$$U((C_t^\ell)_{t\geq 0}) = \sum_{t=0}^\infty \beta^t u(C_t^\ell) \quad \text{where } u(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma > 0, 0 < \beta < 1. \tag{15}$$

Let K_0^ℓ denote the initial capital endowment in $t = 0$ and $\bar{N}_t^\ell > 0$ the labor supplied in period t which is exogenous in our model. As before, let $q_t = \prod_{s=1}^t r_s^{-1}$ denote the discount factor for period t. Defining lifetime labor income $W^\ell := \sum_{t=0}^\infty q_t w_t^\ell \bar{N}_t^\ell$, transfer income T^ℓ as in (9), and profit income Π^ℓ as in (14), the consumer's decision problem reads:

$$\max_{(C_t^\ell)_{t\geq 0}} \left\{ \sum_{t=0}^\infty \beta^t u(C_t^\ell) \;\middle|\; \sum_{t=0}^\infty q_t C_t^\ell \leq r_0 K_0^\ell + W^\ell + \Pi^\ell + T^\ell,\ C_t^\ell \geq 0\ \forall t \geq 0 \right\}.$$

[2] The general version of GHKT allows for γ to be time- and state-dependent. Here, we assume that it is constant, as they do in their numerical simulations, too.

At equilibrium, consumption C_t^ℓ in region $\ell \in \mathbb{L}$ is given by a constant share of world consumption $\bar{C}_t := \sum_{\ell\in\mathbb{L}} C_t^\ell$ each period $t \geq 0$, i.e.,

$$C_t^\ell = \mu^\ell \bar{C}_t = \frac{r_0 K_0^\ell + W^\ell + \Pi^\ell + T^\ell}{\sum_{k\in\mathbb{L}} \left(r_0 K_0^k + W^k + \Pi^k + T^k\right)} \bar{C}_t. \tag{16}$$

The evolution of aggregate consumption is determined by the Euler equation

$$\bar{C}_{t+1} = (\beta r_{t+1})^{\frac{1}{\sigma}} \bar{C}_t \tag{17}$$

and must satisfy the transversality condition

$$\lim_{T\to\infty} \beta^T u'(\bar{C}_t) \bar{K}_{T+1} = 0 \tag{18}$$

where $\bar{K}_t$ is the aggregate world capital stock in period t.

1.5 Market clearing

As labor supply is immobile across regions, the labor market clearing condition for region ℓ in period t reads

$$\sum_{i\in\mathbb{I}_0} N_{i,t}^\ell \overset{!}{=} \bar{N}_t^\ell. \tag{19}$$

By contrast, capital, exhaustible resources, and final output can freely be traded across countries. Letting $\bar{K}_t > 0$ denote the world capital stock in period t, market clearing on the global capital market requires

$$\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_0} K_{i,t}^\ell \overset{!}{=} \bar{K}_t \qquad \forall t \geq 0. \tag{20}$$

The market clearing condition for resource $i \in \mathbb{I}_x$ in period t is $\sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell,s} \overset{!}{=} \sum_{\ell\in\mathbb{L}} X_{i,t}^\ell$. Summing over all countries, production inputs must satisfy the world exhaustible resource constraint

$$\sum_{t=0}^{\infty} \sum_{\ell\in\mathbb{L}} X_{i,t}^\ell \leq R_{i,0} \qquad \forall i \in \mathbb{I}_x. \tag{21}$$

Here, $R_{i,0} := \sum_{\ell\in\mathbb{L}} R_{i,0}^\ell$ denotes the global initial stock of resource $i \in \mathbb{I}_x$. As the Hotelling rule (7) makes resource firms indifferent between the timing of extraction, the amount $X_{i,t}^{\ell,s}$ extracted in a particular region and period is, in general, indeterminate.

Finally, denoting world consumption by $\bar{C}_t$ as before, the world capital stock evolves as

$$\bar{K}_{t+1} = \sum_{\ell\in\mathbb{L}} Y_t^\ell - \bar{C}_t - \sum_{i\in\mathbb{I}_x} c_i \sum_{\ell\in\mathbb{L}} X_{i,t}^\ell \qquad \forall t \geq 0. \tag{22}$$

Equation (22) can be interpreted as a market clearing condition for final output.

1.6 Equilibrium

For $t \geq 0$, define the productivity vector $\mathbf{Q}_t := (Q^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$ and labor supply $\mathbf{N}^s_t := (\bar{N}^\ell_t)_{\ell\in\mathbb{L}}$. The sequences $(\mathbf{Q}_t)_{t\geq 0}$ and $(\mathbf{N}^s_t)_{t\geq 0}$ are exogenously given in our model. Writing $\mathbf{Y}_t := (Y^\ell_t)_{\ell\in\mathbb{L}}$, $\mathbf{E}_t := (E^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}}$, $\mathbf{K}_t := (K^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$, $\mathbf{N}_t := (N^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$, $\mathbf{X}_t := (X^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x}$, $\mathbf{S}_t := (S_{t,1}, S_{t,2})$, $\mathbf{w}_t := (w^\ell_t)_{\ell\in\mathbb{L}}$, $\mathbf{p}_t := (p^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}}$, $\mathbf{v}_t := (v_{i,t})_{i\in\mathbb{I}_x}$, an aggregate equilibrium is a sequence $\xi = (\xi_t)_{t\geq 0}$ defined for each $t \geq 0$ as

$$\xi_t = (\mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, r_t, \mathbf{w}_t, \mathbf{p}_t, \mathbf{v}_t, \tau_t, \mathbf{S}_t, \bar{C}_t, \bar{K}_{t+1}) \tag{23}$$

which is consistent with the production technologies and optimality conditions (1)–(6) of producers, the Hotelling condition (7), the market clearing conditions (19), (20), (22) for labor, capital, and output, the global resource constraint (21), and climate conditions (10)–(13) as well as the Euler equation (17) and the transversality condition (18). The term aggregate is used because ξ_t only involves aggregate consumption $\bar{C}_t$ but not its distribution across regions.

In the simulation study to be presented in this paper, two cases are of particular interest. First, the *laissez-faire equilibrium* $\xi^{\mathrm{LF}} = (\xi^{\mathrm{LF}}_t)_{t\geq 0}$. This equilibrium represents the case where there is no attempt to correct market outcomes by imposing a climate tax. Due to the presence of a climate externality, this solution fails to be Pareto-optimal.

Second, the *efficient equilibrium* $\xi^{\mathrm{eff}} = (\xi^{\mathrm{eff}}_t)_{t\geq 0}$ which maximizes utility of a fictitious world representative consumer and fully corrects the inefficiency of the laissez-faire solution. Along the efficient equilibrium, taxes are determined by the Pigouvian solution

$$\tau_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \left(\phi_L + (1-\phi_L)\phi_0.(1-\phi)^n \right) \sum_{\ell\in\mathbb{L}} \gamma^\ell Y^\ell_{t+n}. \tag{24}$$

The climate tax determined by (24) is called the efficient tax policy and is denoted $\tau^{\mathrm{eff}} = (\tau^{\mathrm{eff}}_t)_{t\geq 0}$. If the efficient solution follows a balanced growth path on which output and consumption grow at constant and identical rate $g \geq 0$, (24) takes the simpler form

$$\tau^{\mathrm{eff}}_t = \bar{\tau}^{\mathrm{eff}} \sum_{\ell\in\mathbb{L}} \gamma^\ell Y^\ell_t, \quad \bar{\tau}^{\mathrm{eff}} := \frac{\phi_L}{1-\beta(1+g)^{1-\sigma}} + \phi_0 \frac{1-\phi_L}{1-\beta(1+g)^{1-\sigma}(1-\phi)}. \tag{25}$$

Thus, on a balanced growth path, the optimal tax is a constant share $\bar{\tau}^{\mathrm{eff}}$ of world output weighted by the damage parameters γ^ℓ.

As the aggregate equilibrium solution (23) does not specify disaggregated consumption in each region, it is independent of the transfer policy $\theta = (\theta^\ell)_{\ell\in\mathbb{L}}$. Once such a transfer policy is specified, the consumption vector $C_t = (C^\ell_t)_{\ell\in\mathbb{L}}$ and the supporting transfers $(T^\ell)_{\ell\in\mathbb{L}}$ can be determined by (16) and (9). The main advantage of determining an aggregate allocation first is that the equilibrium equations give rise to a forward-recursive structure which greatly simplifies their computation.

2 Solving the Model

This section develops an algorithm to compute the aggregate equilibrium sequence $(\xi_t)_{t\geq 0}$ defined in (23) under alternative specifications of the climate tax policy $(\tau_t)_{t\geq 0}$. We confine attention to policies where the tax in period t is determined endogenously as

$$\tau_t = \bar{\tau} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^\ell. \tag{26}$$

Specification (26) induces the laissez-faire equilibrium by setting $\bar{\tau} = 0$ and the (approximated) efficient solution for $\bar{\tau} = \bar{\tau}^{\text{eff}}$ defined as in (25).[3] One can also choose other values for $\bar{\tau}$ and it is also possible to specify the sequence $(\tau_t)_{t\geq 0}$ exogenously.

To reduce the number of equilibrium conditions, we first perform a few simple substitutions. First, substitute (10) using (11) and (12) into (13) and define $\phi_Z := \phi_L + (1-\phi_L)\phi_0$ to obtain climate damage in region ℓ as a function

$$D_t^\ell = \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1}) := 1 - \exp\left\{ -\gamma^\ell \left(\phi_Z \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell + S_{1,t-1} + (1-\phi) S_{2,t-1} - \bar{S} \right) \right\}. \tag{27}$$

Substituting (27) into (1) permits final output in region ℓ at time t to be written as

$$Y_t^\ell = (1 - \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1})) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}). \tag{28}$$

Making the same substitution, the first order conditions (2) of the final sector become

$$\begin{aligned}
(1 - \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1})) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) &= r_t && \text{(29a)} \\
(1 - \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1})) Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) &= w_t^\ell && \text{(29b)} \\
(1 - \hat{D}^\ell(\mathbf{X}_t, \mathbf{S}_{t-1})) Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) &= p_{i,t}^\ell \quad \forall i \in \mathbb{I}. && \text{(29c)}
\end{aligned}$$

In the following computations, it will be more convenient to use (28) and (29) instead of the original equations (1) and (2).

2.1 Simulation parameters

The simulation fixes the set of regions $\mathbb{L}$, energy sectors $\mathbb{I}$, and exhaustible resources $\mathbb{I}_x \subset \mathbb{I}$, assuming that $0 < L := |\mathbb{L}|$ and $1 \leq I_x := |\mathbb{I}_x| \leq I := |\mathbb{I}|$. Given the sectoral structure, one needs to specify the functional forms of the production functions $(F_i)_{i \in \mathbb{I}_0}$ and exogenous sequences for labor supply $(\mathbf{N}_t^s)_{t\geq 0}$ and productivity $(\mathbf{Q}_t)_{t\geq 0}$. Further,

[3] To evaluate the accuracy of this approximation in our simulations, we (a) verify that output and consumption both converge to a balanced growth path and (b) recompute optimal taxes based on the original formula (24) using the series of output and consumption from our simulation. The differences between both tax rates are sufficiently small, notably during the initial simulation periods when the economy still converges to the balanced path such that computing taxes based on (26) is fully justified.

values respecting above's sign restrictions must be assigned to the parameters (β, σ) describing consumer behavior, extraction costs $(c_i)_{i\in\mathbb{I}_x}$, climate parameters (ϕ, ϕ_0, ϕ_L), and damage parameters $(\gamma^\ell)_{\ell\in\mathbb{L}}$. Finally, we fix initial values for the climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$, stocks of global resources $(R_{i,0})_{i\in\mathbb{I}_x}$, and aggregate capital $\bar{K}_0 > 0$ and choose the value $\bar{\tau}$ for the climate tax policy (26).

2.2 Forward-recursive structure

Our numerical algorithm exploits the forward-recursive structure of the model to determine the vector ξ_t defined in (23) as a function of ξ_{t-1} and exogenous variables. To make this idea precise, partition the equilibrium vector as $\xi_t = (\xi_t^1, \xi_t^2)$ where

$$\xi_t^1 := (\mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, r_t, \mathbf{w}_t, \mathbf{p}_t, \mathbf{v}_t, \tau_t) \tag{30a}$$

$$\xi_t^2 := (\mathbf{S}_t, \bar{C}_t, \bar{K}_{t+1}). \tag{30b}$$

Note that ξ_t^1 is an $M := 4L(I+1) + (L+1)I_x + 2$-dimensional vector taking values in

$$\Xi^1 := \mathbb{R}^L_{++} \times \mathbb{R}^{LI}_{++} \times \mathbb{R}^{L(I+1)}_{++} \times \mathbb{R}^{L(I+1)}_{++} \times \mathbb{R}^{LI_x}_{++} \times \mathbb{R}_{++} \times \mathbb{R}^L_{++} \times \mathbb{R}^{LI}_{++} \times \mathbb{V} \times \mathbb{R}_+ \tag{31}$$

where $\mathbb{V} := \prod_{i\in\mathbb{I}_x}[c_i, \infty[\subset \mathbb{R}^{I_x}_+$. Vector ξ_t^2 takes values in the set $\Xi^2 := \mathbb{R}^2 \times \mathbb{R}_{++} \times \mathbb{R}_{++}$. For each period $t \geq 0$, the relevant pre-determined variables are collected in a vector

$$\theta_t := (\mathbf{N}_t^s, \mathbf{Q}_t, \mathbf{v}_{t-1}, \mathbf{S}_{t-1}, \bar{C}_{t-1}, \bar{K}_t) \tag{32}$$

with values in $\Theta := \mathbb{R}^L_{++} \times \mathbb{R}^{L(I+1)}_{++} \times \mathbb{V} \times \mathbb{R}^2 \times \mathbb{R}_{++} \times \mathbb{R}_{++}$. Note that θ_t consist of the exogenous variables $(\mathbf{N}_t^s, \mathbf{Q}_t)$ and a number of endogenous variables from ξ_{t-1}.

Given $\theta_t \in \Theta$, the main step in our algorithm is to determine ξ_t^1 by simultaneously solving equations (3), (4), (5), (6), (7), (19), (20), (26), (28), and (29). Note that these conditions constitute a system of $LI_x + 3LI_x + L(I - I_x) + 2L(I - I_x) + I_x + L + 1 + 1 + L + L(I+2) = 4L(I+1) + (L+1)I_x + 2 = M$ non-linear equations that can potentially be solved to obtain a unique vector $\xi_t^1 \in \Xi^1$.

To formalize this problem, define the mapping $\Phi : \Xi^1 \times \Theta \longrightarrow \mathbb{R}^M$ such that given $\theta_t \in \Theta$, ξ_t^1 solves equations (3)-(7), (19), (20), (26), (28), and (29) if and only if $\Phi(\xi_t^1, \theta_t) = 0$. For example, if $1 \in \mathbb{I}_x$, the first component function $\Phi_1 : \Xi^1 \times \Theta \longrightarrow \mathbb{R}$ defined by the first entry of equation (3) would be $\Phi_1(\xi_t^1, \theta_t) = E^1_{1,t} - Q^1_{1,t}F_1(K^1_{1,t}, N^1_{1,t}, X^1_{1,t})$.

Given the pre-determined variables θ_t and the solution ξ_t^1, ξ_t^2 can be determined directly by equations (11), (17), and (22). These equations define a function $\Psi : \Xi^1 \times \Theta \longrightarrow \Xi^2$ which determines $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$. Determining ξ_t^1 and ξ_t^2 in this fashion based on predetermined variables collected in θ_t defines one iteration step of our model.

2.3 Computational algorithm

The following sequential structure illustrates our computational algorithm for an iteration of the model of length $t^{\max} > 0$.

Step 1: Initialization for $t = 0$:[4]

(a) Choose candidate values for initial consumption $\bar{C}_{-1} > 0$ and initial resource prices $\mathbf{v}_{-1} = (v_{i,-1})_{i \in \mathbb{I}_x} \in \mathbb{V}$. If $R_{i,0} = \infty$, set $v_{i,-1} = c_i$, otherwise $v_{i,-1} > c_i$.

(b) Use these values together with $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ and $\bar{K}_0 > 0$ to determine the endogenous part of θ_0. Set $t = 0$.

Step 2: Iteration for $0 \leq t \leq t^{\max}$:

(a) Compute θ_t using $(\mathbf{N}_t^s, \mathbf{Q}_t)$ and the relevant endogenous variables from $t-1$.

(b) Compute ξ_t^1 by solving $\Phi(\xi_t^1, \theta_t) = 0$ as outlined above.

(c) Compute $\xi_t^2 = \Psi(\xi_t^1, \theta_t)$ as outlined above and check the following conditions:

- If $\bar{K}_{t+1} < 0$, return to **Step 1** and decrease $\bar{C}_{-1}$.
- If $\bar{C}_t < \bar{C}_t^{\text{crit}}$, return to **Step 1** and increase $\bar{C}_{-1}$.
- Otherwise, increase t by 1.

Step 3: Verification of resource constraints in $t = t^{\max}$:

(a) For all $i \in \mathbb{I}_x$, compute $R_{i,t^{\max}+1} := R_{i,0} - \sum_{t=0}^{t^{\max}} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell}$:

- If $R_{i,t^{\max}+1} < 0$, return to **Step 1** and increase $v_{-1,i}$.
- If $R_{i,t^{\max}+1} > R_i^{\text{crit}}$, return to **Step 1** and increase $v_{-1,i}$.

(b) If $0 < R_{i,t^{\max}+1} < R_i^{\text{crit}}$ for all $i \in \mathbb{I}_x$, complete the iteration. ■

Step 2(c) in the previous algorithm requires the specification of a (typically time-dependent) lower bound $\bar{C}_t^{\text{crit}}$ for consumption in period t.[5] The condition $\bar{C}_t > \bar{C}_t^{\text{crit}}$ for all t serves to exclude cases where consumption *implodes*, i.e., converges to zero. This case occurs when initial consumption $\bar{C}_{-1}$ is chosen too small. Conversely, if $\bar{C}_{-1}$ is chosen to large, consumption *explodes*, i.e., grows too fast relative to output. In this case, the condition $\bar{K}_{t+1} > 0$ for all t will eventually be violated. Excluding both cases determines a unique initial value $\bar{C}_{-1}$ for which the equilibrium dynamics are well defined and satisfy the transversality condition (18). These features are well-known for

[4]Specifying initial values for $\mathbf{v}_{-1}$ and $\bar{C}_{-1}$ and computing $\mathbf{v}_0$ and $\bar{C}_0$ using (7) and (17) allows us to cast all computations for $t = 0$ in the same form as for periods $t > 0$. The structure of equations (7) and (17) and the fact that the values $\mathbf{v}_{-1}$ and $\bar{C}_{-1}$ only affect $\mathbf{v}_0$ and $\bar{C}_0$ shows that this approach is mathematically equivalent to assigning initial values to $\mathbf{v}_0$ and $\bar{C}_0$ directly.

[5]Our simulations use $\bar{C}_t^{\text{crit}} = \bar{c}^{\text{crit}} \left(\sum_{\ell \in \mathbb{L}} Y_t^{\ell} - \sum_{i \in \mathbb{I}_x} c_i \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell} \right)$ where $\bar{c}^{\text{crit}}$ is a small number.

the neoclassical growth model in state space form which exhibits saddle-path stability requiring initial consumption to be chosen on the stable manifold of values which converge to the steady state. These features carry over to the present more complicated model. Our numerical approach determines the unique sustainable initial level $\bar{C}_{-1}$ such that $\bar{K}_{t+1} > 0$ and $\bar{C}_t > \bar{C}_t^{\text{crit}}$ for all $t \leq t^{\max} + N^{\text{ahead}}$ for some $N^{\text{ahead}} \geq 0$.[6]

The conditions evaluated in Step 3 concern the world resource constraints (21). Clearly, this condition becomes relevant only for resources $i \in \mathbb{I}_x$ for which $R_{i,0} < \infty$. Suppose this is the case and define $R_{i,t+1} := R_{i,t} - \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell}$ as the world resource stock at the end of period t. For any candidate resource price $\hat{v}_{i,-1}$, the induced sequence $(\hat{R}_{i,t})_{t\geq 0}$ of world resource stocks is strictly decreasing and, therefore, converges to a unique limit $\hat{R}_{i,\infty}$ which is zero at equilibrium. In our simulations, we establish that the sequence $(\hat{R}_{i,t})_{t\geq 0}$ becomes approximately constant within the length of iteration such that $\hat{R}_{i,\infty}$ can be approximated by $\hat{R}_{i,t^{\max}+1}$. We now adjust the initial resource price $\hat{v}_{i,-1}$ until $\hat{R}_{i,t^{\max}+1}$ becomes approximately zero, increasing $\hat{v}_{i,-1}$ when $\hat{R}_{i,t^{\max}+1} < 0$ and decreasing $\hat{v}_{i,-1}$ when $\hat{R}_{i,t^{\max}+1} < 0$. The iteration stops when all terminal resource stocks are (in absolute terms) less than a pre-specified critical value R_i^{crit} which is chosen close to zero. The current value $\hat{v}_{i,-1}$ then approximates the initial equilibrium resource price $v_{i,-1}$.

2.4 Computational details

The key challenge in our algorithm is to determine the vector ξ_t^1 which solves the condition $\Phi(\xi_t^1, \theta_t) = 0$ in Step 2(b). Mathematically, this is a standard fixed point problem which can be solved using standard numerical routines like the Newton-Raphson algorithm, etc. As our simulations are directly implemented in $C++$, however, we designed our own more 'economic' algorithm. Intuitively, this algorithm successively equates marginal products across different markets by reallocating production factors based on the differences between their respective marginal products. We employ a nested market structure where the labor allocation in both regions is determined first for a given allocation of capital and exhaustible resources, then the global capital allocation is determined for a given resource allocation (with labor constantly readjusting) and, finally, the global resource allocation is computed. While potentially inefficient in terms of computational speed, this proved to be a reliable way of computing the solution ξ_t^1. Details are provided in Appendix A.

[6] In fact, to reduce computation time, we choose initial consumption C_{-1} such that $\bar{C}_t > \bar{C}_t^{\text{crit}}$ and $\bar{K}_{t+1} > 0$ holds for all $0 \leq t \leq N^{\text{ahead}} = 10$. Then, in each future period $t > 0$, the value $\bar{C}_t$ delivered by the Euler equation (17) is (slightly) adjusted such that $\bar{C}_{t+n} > \bar{C}_{t+n}^{\text{crit}}$ and $\bar{K}_{t+n} > 0$ holds for all $0 \leq n \leq N^{\text{ahead}}$. Thus, in each period, we adjust consumption to ensure that the consumption-capital dynamics is stable over the next N^{ahead} periods. As these adjustments are small if N^{ahead} is chosen sufficiently large, our approach is equivalent to choosing initial consumption C_{-1} such that the dynamics is stable for all $t \leq t^{\max} + N^{\text{ahead}}$ but turned out to be computationally faster. In addition, one can successively increase the accuracy of the simulations by gradually increasing N^{ahead}.

2.5 Regional consumption and transfers

The aggregate equilibrium (23) computed in the previous sections does not specify regional consumption $\mathbf{C}_t = (C_t^\ell)_{\ell \in \mathbb{L}}$ and the transfers (9) between regions. Computation of these values requires the specification of a transfer policy $\theta = (\theta^\ell)_{\ell \in \mathbb{L}}$ and the initial distribution of capital $(K_0^\ell)_{\ell \in \mathbb{L}}$ and exhaustible resources $(R_{i,0}^\ell)_{\ell \in \mathbb{L}}$ of each type $i \in \mathbb{I}_x$. Once these objects are specified, we need to approximate lifetime labor incomes $(W^\ell)_{\ell \in \mathbb{L}}$ and transfer incomes $(T^\ell)_{\ell \in \mathbb{L}}$ defined as above. For each $\ell \in \mathbb{L}$ define for $N > 0$

$$W_N^\ell := \sum_{t=0}^{N} q_t w_t^\ell N_t^\ell \tag{33}$$

and total discounted tax revenue

$$T_N := \sum_{t=0}^{N} q_t \tau_t \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell . \tag{34}$$

Both sequences $(W_N^\ell)_{N \geq 0}$ and $(T_N)_{N \geq 0}$ are strictly increasing and we verify numerically that they converge sufficiently fast and become nearly constant as $N \to t^{\max}$ where we use $t_{\max} = 50$ in our simulations. This allows us to approximate W^ℓ by $\hat{W}^\ell := W_{t^{\max}}^\ell$ and T^ℓ by $\hat{T}^\ell := \theta^\ell T_{t^{\max}}$.

With these approximations, one can employ (16) to obtain (approximated) consumption in region $\ell \in \mathbb{L}$ as

$$\hat{C}_t^\ell = \hat{\mu}^\ell \bar{C}_t = \frac{r_0 K_0^\ell + \hat{W}^\ell + \Pi^\ell + \hat{T}^\ell}{\sum_{k \in \mathbb{L}} \left(r_0 K_0^k + \hat{W}^k + \Pi^k + \hat{T}^k \right)} \bar{C}_t \quad \forall t \geq 0 \tag{35}$$

with profit incomes Π^ℓ determined by (14) from the initial distribution of exhaustible resources across regions.

3 Calibrating the Model

The world economy is divided into $L = 2$ regions. Region $\ell = 1$ represents the member states of the *'Organization for Economic Cooperation and Development'* and will be called the *OECD countries*. Region $\ell = 2$ comprises the rest of the world and will be referred to as the *NOECD countries*. Our study compares the laissez-faire and the optimal tax policy discussed in the previous sections and determines optimal transfers between OECD and NOECD countries for the transfer policy given in equation (9).

3.1 Main calibration targets

Following GHKT and Acemoglu et al. (2012), each model period represents ten years. The initial simulation period ends in $t = 2015$ and is called the baseline period. Our calibration is based on the following empirical observations which we match in the baseline period:

- The world population share of OECD-countries is currently 18%[7]
- GDP in OECD countries makes up 69% of current world GDP[8]
- OECD countries owned $68,5\%$ of the global capital stock in 2015[9]
- 17% of global crude oil and 10% of natural gas reserves are located in OECD countries.[10]

Further targets which we match in our simulations are introduced below.

3.2 Energy sectors

Each region has $I = 3$ energy sectors. Sector $i = 1$ comprises energy outputs and services derived from *crude oil and natural gas* including all traffic and transportation services based on oil and gas such as motorvehicles, cargo aircrafts, railroad cargo etc. as well as oil refineries which produce petroleum products. Sector $i = 2$ produces energy based on *coal* (anthracite coal and lignite). Its output comprises essentially coal-based power generation and heat. Sector $i = 3$ subsumes all energy goods and services which do not generate emissions. For simplicity, we assume that production in this sector is exclusively based on renewable energy sources.[11] With our previous notation we thus have $\mathbb{I} = \{1, 2, 3\}$ and $\mathbb{I}_x = \{1, 2\}$.

[7]The World Bank. World Development Indicators (2015). *Total Population*. Available from `http://data.worldbank.org/indicator/SP.POP.TOTL`

[8]The World Bank. World Development Indicators (2015). *GDP(current US-$)*. Retrieved from `http://data.worldbank.org/indicator/NY.GDP.MKTP.CD`

[9]Berlemann & Wesselhoeft (2014) reported estimates of capital stocks for 103 countries for the period 1970-2011 using the perpetual inventory method. Their estimates imply a world capital stock of 64,499 Billion US-$\$_{2000}$ of which 44,208 Billion US-$\$_{2000}$ (68,5%) is located in OECD-member states.

[10]According to the German Federal Institute for Geosciences and Natural Resources (BGR), current global crude oil reserves are 219 Gt (Giga tons) of which 181 Gt are located in NOECD-countries. Current natural gas reserves are 197,8 Trn.m^3, of which 178 Trn.m^3 are in NOECD-countries.

[11]As nuclear energy production would also be included in sector 3, this abstracts from the fact that uranium is an exhaustible resource, too. This seems justified, however, because the existing stocks of uranium are abundant relative to fossil reserves.

The production technologies in (1),(3), and (5) are specified as follows. The production function in the final sector takes the CES-form

$$F_0(K,N,(E_i)_{i\in\mathbb{I}}) = \Big[\alpha_{0,K}K^{\varrho_0} + \alpha_{0,N}N^{\varrho_0} + (1-\alpha_{0,K}-\alpha_{0,N})G((E_i)_{i\in\mathbb{I}})^{\varrho_0}\Big]^{\frac{1}{\varrho_0}} \tag{36}$$

where $\alpha_{0,K} > 0$, $\alpha_{0,N} > 0$, $\alpha_{0,K}+\alpha_{0,N} < 1$, and $\varrho_0 < 1$. Here, G is an aggregator function which aggregates the different energy types to a composite energy input $G((E_i)_{i\in\mathbb{I}}$ which will be specified below.

Exhaustible energy sectors $i \in \mathbb{I}_x = \{1,2\}$ use a technology of the form

$$F_i(K,N,X) = X^{\alpha_{i,X}}\Big[\alpha_{i,K}K^{\varrho_i} + (1-\alpha_{i,K})N^{\varrho_i}\Big]^{\frac{1-\alpha_{i,X}}{\varrho_i}} \tag{37}$$

where $0 < \alpha_{i,X} < 1$, $0 < \alpha_{i,K} < 1$, and $\varrho_i < 1$. This specification allows us to vary the elasticity of substitution $\frac{1}{1-\varrho_i}$ between labor and capital while maintaining our earlier assumption that exhaustible resources constitute an essential input to production.

Finally, the technology used by the clean sector $i = 3$ is again of the CES-type

$$F_3(K,N) = \Big[\alpha_{3,K}K^{\varrho_3} + (1-\alpha_{3,K})N^{\varrho_3}\Big]^{\frac{1}{\varrho_3}} \tag{38}$$

where $0 < \alpha_{3,K} < 1$ and $\varrho_3 < 1$.

In our benchmark parametrization we set $\varrho_i = 0$ in (36), (37), and (38) inducing a Cobb-Douglas technology in each sector which allows us calibrate the α parameters based on observed cost shares of production factors.

With the previous interpretation in mind, we set $\alpha_{0,K} = 0.3$ in (36) which is a value commonly used in the literature. We used world input-output tables constructed in Timmer et al. (2015) to calculate the cost share of energy in final output production and accordingly set $\alpha_{0,E} = 0.075$ implying a cost share of labor equal to 62.5%.[12]

For sector $i = 1$, we computed cost shares corresponding to $\alpha_{1,K} = 0.85$ and $\alpha_{1,X} = 0.33$ based on data from the U.S Bureau of Economic Analysis (2007).[13]

As sectors $i = 2,3$ mainly produce electricity, we can base our parameter choices on the nominal electricity generation costs for Germany reported in Hillebrand (1997).[14] For

[12]The cost share of energy is higher than the value of 4% used in GHKT. However, as energy inputs to final production represent intermediate goods and services produced from fossil resources in our model rather than exhaustible resources as in GHKT, our higher share of energy costs reflects the value added at the energy production stage.

[13]The data highlights inter-sectoral linkage for 389 industries/commodities for the United States that can be aggregated to specific sectors which are relevant for this study, for instance "Refineries" or "Transportation". Especially for these three isolated industry groups, which represent our energy type $i = 1$ "oil and gas based energy goods and services", we then calculated the parameters representing the cost shares for capital, labor and resources. Additional details are available upon request.

[14]Drawing on data sources for different countries we presume that the underlying technologies are similar enough such that the resulting cost shares for capital and labor are roughly the same.

coal-fired power plants, this study delivers production elasticities $\alpha_{2,K} = 0.69$ for capital and $\alpha_{2,X} = 0.26$ for coal, respectively, which we use directly for sector $i = 2$.

It seems more difficult to choose these parameters for sector $i = 3$ which comprises all emissions-free technologies including nuclear power generation. The share of capital costs for nuclear power plants in Hillebrand (1997) is $\alpha_{3,K} = 0.7$. While nuclear energy makes up a large part of emissions free energy production, sector $i = 3$ also includes renewable energies like wind or solar power for which Loeschel & Otto (2009) report an even higher share of capital costs. For this reason, we choose a slightly higher capital share setting $\alpha_{3,K} = 0.75$. The previous two values are also in line with the general observation made by the Department of Energy & Climate Change (2013) that electricity generated from nuclear as well as wind and hydro power plants is relatively more capital intensive compared to conventional fossil-based electricity or thermal power generation.

The function G in (36) which aggregates energy goods and services used in final output production is determined in two steps. First, the following function G_2 aggregates outputs produced in sectors $i = 2$ and 3 (electricity and heat) to an intermediate composite

$$EL_t^\ell = G_2(E_{2,t}^\ell, E_{3,t}^\ell) := \left[\kappa_2 (E_{2,t}^\ell)^{\rho_2^E} + (1 - \kappa_2)(E_{3,t}^\ell)^{\rho_2^E}\right]^{\frac{1}{\rho_2^E}}. \tag{39}$$

The parameter ρ_2^E determines the elasticity of substitution between CO_2-intensive and clean electricity. Setting $\rho_2^E = 0.6$ in (39) we follow Loeschel & Otto (2009) who report an elasticity of substitution equal to 2.5. We also let $\kappa_2 = 0.5$ which is in line with the observations in GHKT who choose a relative price between dirty and clean electricity generation equal to unity.

A second function G_1 then aggregates the electricity composite EL_t^ℓ with oil and gas-based energy services $E_{1,t}^\ell$ produced in sector $i = 1$ to the final energy composite

$$E_t^\ell = G(E_{1,t}, E_{2,t}, E_{3,t}) = G_1(E_{1,t}^\ell, EL_t^\ell) := \left[\kappa_1 (E_{1,t}^\ell)^{\rho_1^E} + (1 - \kappa_1)(EL_t^\ell)^{\rho_1^E}\right]^{\frac{1}{\rho_1^E}}. \tag{40}$$

Using industry data from the U.S Bureau of Economic Analysis (2007), we set $\kappa_1 = 0.3818$ in (40), which corresponds to the cost of electricity and heat production relative to the cost of transportation per unit of GDP. There is some considerable degree of freedom to restrict the parameter ρ_1^E in (40) which determines the elasticity of substitution between electricity and fossil fuel.[15] We choose a moderately positive value of $\rho_1^E = 0.2$. This also ensures that energy produced by sector $i = 1$ is not an essential input to final production which allows for the model to have a well-defined balanced growth path.

[15]A recent study by Stern (2012) reports values for this elasticity ranging from -3.265 to 8.922.

3.3 Labor supply and productivity growth

The initial distribution of labor supply $\mathbf{N}_0^s = (N_0^\ell)_{\ell\in\mathbb{L}}$ is chosen as $N_0^1 = 0.18$ and $N_0^2 = 0.82$ based on relative population sizes of the two regions with world labor supply normalized to unity. Growth in our model enters via exogenous labor-augmenting change due to which the sequence $(\mathbf{N}_t^s)_{t\geq 0}$ grows at constant rate $g > 0$ in each component. Setting $g = 0.16$ implies an annual growth rate of productivity equal to 1.5% which is a conservative estimate in line with GHKT and most studies of the business cycle.

Differences in productivity are captured by region-specific total factor productivities $Q_{i,t}^\ell \equiv Q_i^\ell$ which are constant over time but allowed to differ across regions $\ell \in \mathbb{L}$ and sectors $i \in \mathbb{I}_0$. For the final sector $i = 0$, we chose relative productivities to match the observed GDP shares reported above while their absolute levels induce a plausible world output of about 700 trillion \$ in the initial modelling period which is also used in GHKT. For energy sectors $i \in \mathbb{I}$, the relative sizes of productivities are chosen to obtain a plausible energy mix in both regions along the laissez-faire equilibrium.[16]

3.4 Resource sectors

Global extraction costs of coal reported by the International Energy Agency (2010, p. 212) average to 43 \$ per ton of coal. This value corresponds to a parameter choice $c_2 = 0.000043$ in our model which we choose directly in our simulations.

For oil and gas, however, empirically measured extraction costs differ considerably across regions and, in addition, often represent short term operating costs not including long-term costs for capital, etc.[17] For this reason, our calibration strategy is to determine the cost parameter c_1 such that the induced initial price $v_{1,0}$ of the composite oil/gas resource at the laissez-faire equilibrium is consistent with empirically observed prices. Using data from the World Bank (2015a) from 2002-2016, we compute the average price for crude oil and natural gas weighted by their respective shares of total global reserves. This gives an initial price of 49.8 \$/bbl corresponding to 356 \$/t which we match in our initial simulation period by setting $c_1 = 0.00025568$.

According to data from the Federal Institute for Geosciences and Natural Resources (2015), global fossil resources include 219 Gt of crude oil and 179 Gt of natural gas.[18]

[16]Data from the International Energy Agency (2014) report that total primary energy supply in OECD countries decomposes into a share of 62% for oil and natural gas, 18% for coal, and 20% for clean energies. For NOECD countries, the corresponding shares are 46%, 35%, and 18% which we match in our baseline period.

[17]For instance, short term operating costs of crude oil extraction reported in the World Economic Outlook 2015 by the International Monetary Fund (2015) range from 4.4 \$/bbl (31.4 \$/t) in Kuwait up to 12 \$/bbl (85.7 \$/t)in Venezuela.

[18]The figures are based on the concept of 'proven reserves' which allows for changes in resource prices but assumes that firms can fully exploit these resources without any change in the applied technology.

In accordance with the stylized facts reported above, we fix the initial stock of resource 1 (oil/gas) in OECD countries at $R_{1,0}^1 = 56$ Gt and $R_{1,0}^2 = 342$ Gt in NOECD countries. For resource 2 (coal), we adopt the same arguments as in GHKT to assume that there is no scarcity rent on coal such that the stock of coal is not exploited. Formally, this corresponds to $R_{2,0} = \infty$ in our simulations which can be justified by a backstop technology which will replace current coal usage in the future. This restriction also implies $v_{2,t} = c_2$ for all $t \geq 0$. As coal extraction generates zero profits, the world distribution of coal reserves is irrelevant. A modification of these restrictions is explored in Section 5 where there is no backstop technology implying that the initial stock of coal is finite.

3.5 Climate dynamics and damages

As our model of the Carbon cycle (11) and the damage function (13) are identical to GHKT, we also use their parameter values setting $\phi_0 = 0.393$, $\phi_L = 0.2$, $\phi = 0.0228$, and $\gamma_1 = \gamma_2 = 5.3 \times 10^{-5}$ in the benchmark case with homogeneous climate damages. Regional differences in γ^ℓ will be explored below. The pre-industrial CO_2-level is $\bar{S} = 581$ and the initial values for permanent and non-permanent CO_2 are $S_{1,-1} = 705$ and $S_{2,-1} = 123$ GtC. For these values, global carbon concentration in the initial simulation period matches the empirically observed CO_2-concentration of 853 GtC in 2015 obtained from the Earth System Research Laboratory (2016).

The carbon content ζ_i of resources $i \in \mathbb{I}_x$ are specified as follows. For $i = 1$, we average the emission factors of crude oil and natural gas weighted by the respective shares of global reserves. This yields a specific carbon content of $\zeta_1 = 0.74681$, corresponding to 746.8 KgC/t. For $i = 2$, we average the emission factors of lignite and anthracite weighted by the respective shares of total global coal reserves. This gives a specific emission content of $\zeta_2 = 0.55854$ corresponding to 558.5 KgC/t.[19]

3.6 Consumption sector

Restricting consumer utility as in (15), we choose $\sigma = 1$ which gives a logarithmic utility function. The annual discount factor is $\beta = 0.985$, so for the model we set $\beta = 0.985^{10}$. These values are identical to the ones used by GHKT in their benchmark scenario. Further, in accordance with the stylized facts reported above, consumers in region 1 own 68.5% of the initial world capital stock. The latter is chosen to obtain a capital-to

[19] Our specific carbon content of 746.8 KgC/t oil and gas, is slightly lower than a carbon content of 844 KgC/t oil given in GHKT. This is due to our additional consideration of natural gas which has a lower specific carbon content compared to oil. This small deviation holds also in the case of coal, since we choose a weighted average of anthracite and lignite with corresponding different carbon contents, while GHKT set their carbon content of coal equal to the value of anthracite. Expressed in carbon units, we assume 560 KgC/t of coal, while GHKT assume 716 KgC/t coal.

labor ratio close to its long-run value along the laissez-faire equilibrium. This avoids a transitory effect due to initial capital adjustment dynamics.

Table 1 summarizes the parameters choices motivated above which are used in the benchmark simulation.

Simulation parameters			
Final sector			
$\rho_0 = 0$	$\alpha_{0,K} = 0.3$	$\alpha_{0,N} = 0.625$	$\alpha_{0,E} = 0.075$
$\rho_1^E = 0.2$	$\rho_2^E = 0.6$	$\kappa_1 = 0.3818$	$\kappa_2 = 0.5$
$Q_0^1 = 3.1$	$Q_0^2 = 0.7$		
Energy sectors			
$\rho_1 = 0$	$\alpha_{1,K} = 0.85$	$\alpha_{1,X} = 0.33$	$Q_1^1 = 96,\ Q_1^2 = 19$
$\rho_2 = 0$	$\alpha_{2,K} = 0.69$	$\alpha_{2,X} = 0.26$	$Q_2^1 = 3.5, Q_2^2 = 8.2$
$\rho_3 = 0$	$\alpha_{3,K} = 0.75$		$Q_3^1 = 40,\ Q_3^2 = 50$
Resource sectors			
$c_1 = 0.000225568$	$c_2 = 0.000043$		
Climate parameters			
$\zeta_1 = 0.74681$	$\zeta_2 = 0.55854$	$\bar{S} = 581$	$\phi_L = 0.2$
$\phi_0 = 0.393$	$\phi = 0.0228$	$\gamma_1 = 0.000053$	$\gamma_2 = 0.000053$
Consumption sector			
$\beta = 0.985^{10}$	$\sigma = 1$	$g = 0.16$	
Initial values			
$K_0 = 0.15$	$R_{1,0} = 398$	$S_{1,-1} = 705$	$S_{2,-1} = 123$

Table 1: Parameter values used in benchmark simulation.

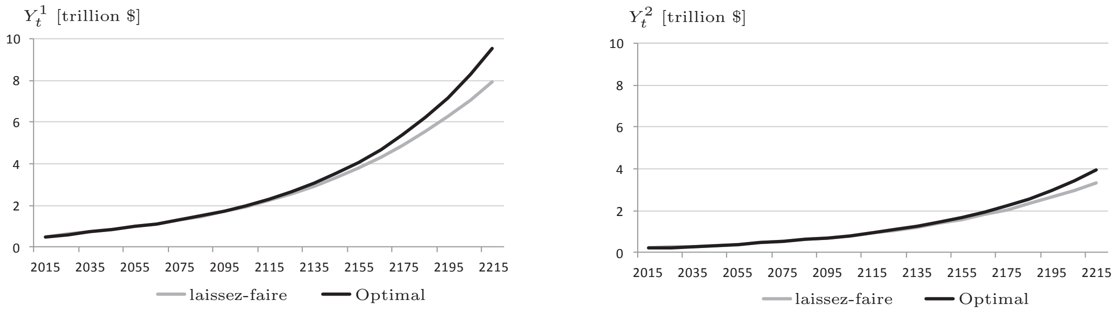
4 Simulation results

Using the parametrization listed in Table 1, the simulation results[20] presented in this section compare the optimal and laissez faire equilibrium at the final stage, the energy stage, the resource stage, and the climate stage. We also study the direction and size of optimal transfers between the two regions based on the Pareto-improving transfer policy developed in Hillebrand & Hillebrand (2017). There, we also allow for heterogeneities in climate damages which may be more severe in NOECD countries. The latter region represents the less-developed countries in our study which tend to be more vulnerable to climate damages than developed countries, see World Bank (2010).

4.1 Final output stage

Figure 1 compares production output Y_t^ℓ in both regions under laissez-faire and optimal taxation. For the initial baseline period, our model predicts a world GDP $Y_t = Y_t^1 + Y_t^2$

[20] All simulation results can be downloaded at `http://www.marten-hillebrand.de/research/`



(a) OECD

(b) NOECD

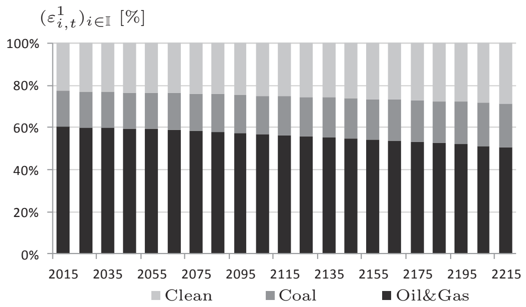
Figure 1: Final production in each region: Optimal vs. laissez-faire solution.

of 707 trillion US$ at the laissez-faire solution and 702 trillion under optimal taxation of which roughly 70% is produced in OECD countries. These values closely match the empirical targets reported above. In the following periods until $t = 2055$, output in both regions continues to be higher under laissez-faire. Quantitatively, the climate tax reduces GDP in OECD countries by about -0.5% in both $t = 2015$ and 2025 and by about -0.6% in $t = 2035$ and 2045 relative to laissez-faire. For NOECD countries, relative output drops by about -0.9% in $t = 2015$ and 2025 and by -1.2% in $t = 2035$ and 2045. This implies a decrease of world output by about -0.6% in t=2015 and 2025 and -0.8% in t=2035 and 2045 under optimal taxation relative to the laissez-faire solution. From $t = 2055$ onwards, this effect reverses and output in both regions is relatively higher under optimal taxation in all periods thereafter with the gap continuously increasing. After 100 years, in $t = 2115$, optimal taxation leads to global output already 2% higher than without taxation and almost 20% higher after 200 years in $t = 2215$.

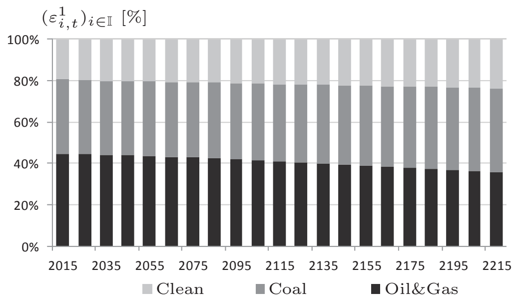
Furthermore, under optimal taxation the economy quickly converges to a balanced growth path (BGP) along which output and also consumption and capital grow at approximately constant and identical rates of about 15.6% corresponding to an annual growth rate of almost 1.5%. The emissions tax therefore allows the economy to sustain a positive growth rate which is only slightly smaller than the growth rate $g = 0.16$ of technological progress due to the productivity-diminishing effect of climate damage and the presence of exhaustible resources and extraction costs. By contrast, annual growth along laissez-faire is increasingly harmed by climate damages and becomes less than 1.4% after $t = 2115$ and less than 1.17% after $t = 2215$. For longer time horizons, these losses becomes even more severe. Summarizing, these results support the intuition that climate policies come at some initial costs which are however negligible compared to the gains in the long-run while laissez-faire leads to dramatic losses in output and growth.

4.2 Energy stage

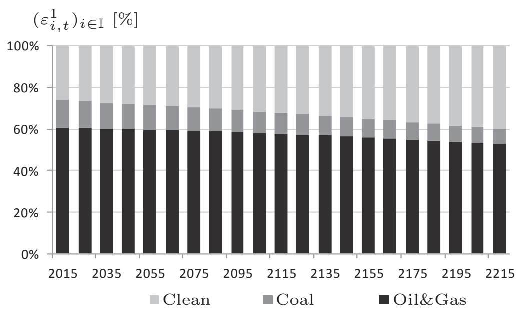
Our specification of production sectors allows us to study how climate policy induces sectoral changes at the energy level. We employ two measures to quantify these changes. The first one is the *energy mix* which measures the percentage share that each energy sector contributes to the total value of energy production (the latter measured in units of final output). Figure 2 compares the energy mix in OECD and NOECD countries under laissez-faire and optimal taxation. By construction of our parameter set, the initial



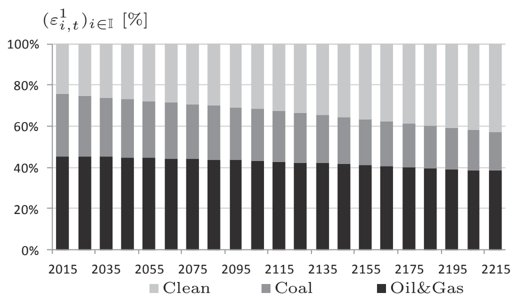
(a) OECD countries



(b) NOECD countries



(c) OECD countries



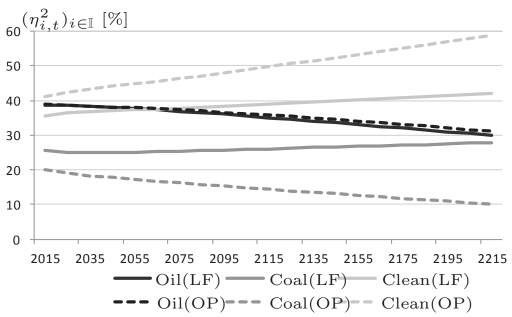
(d) NOECD countries

Figure 2: Energy mix in both regions under laissez faire and optimal taxation.

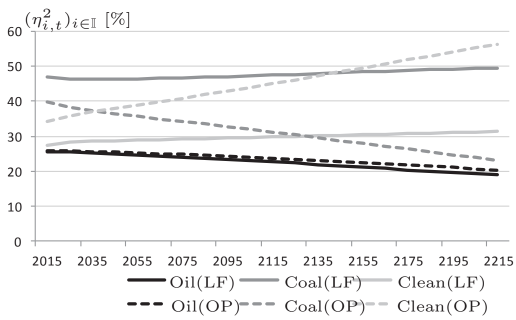
values match the empirically observed energy shares reported above (cf. footnote16) at the laissez faire equilibrium. For this scenario, fossil fuels (coal, oil, and gas combined) initially dominate energy production in both regions while emission free technologies make up only 22% in the OECD and 19% in the NOECD. While economic activity in both regions depends strongly on fossil fuels, this dependence is more biased towards oil and gas in OECD countries which make up 77% of total fossil fuel consumption. For NOECD countries in which coal usage is more dominant, the corresponding share is only 54%. Furthermore, OECD countries are responsible for roughly 70% of global energy usage which is equivalent to the OECD's share of global GPD reported above. Formally, this is a direct consequence of the Cobb-Douglas specification for final output production in both regions. Over the course of the next two-hundred years, our model predicts a gradual decline of oil consumption which is gradually and almost evenly replaced by coal and clean energy. Thus, even under laissez-faire clean energies acquire a higher share over time due to the increased scarcity of oil.

Under the optimal climate policy, the initial energy mix and shares of global energy consumption are roughly the same as under laissez faire. Over time, however, the carbon tax induces a substitution from fossil fuels to clean technologies. After 100 years, clean energy produces 32% (33%) of total energy supply in the OECD (NOECD), which is an increase of 23% (34%) compared to the initial state. Interestingly, the value share of oil is larger compared to laissez faire in both regions. Thus, the decarbonization of energy supply under the optimal policy is primarily driven by a reduction in coal consumption. This also confirms the general insight that coal is the main driver of climate change also emphasized in GHKT.

Our second measure of structural change quantifies the employment effects at the energy stage which are measured by the percentage share of energy sector i of total employment in domestic energy sectors. Figure 3 compares the employment shares under laissez faire



(a) OECD



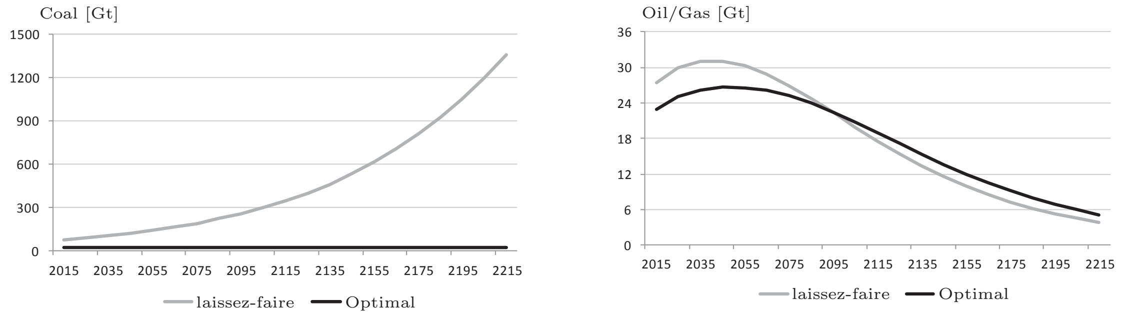
(b) NOECD

Figure 3: Employment shares in energy sectors: Optimal vs. laissez-faire solution.

(solid lines) and optimal policy (dashed lines) for both regions. One observes that in both policy scenarios, relative employment in the oil-based energy sector declines over time. Under laissez faire, this reduced share is evenly absorbed by coal and clean energy production which both increase their share over time. Thus, even under laissez faire the fraction of people working in clean energy production increases over time, a finding qualitatively similar to the previous figure. Comparing laissez faire and the optimal policy, we find that employment shares in the oil sector are largely unaffected by taxation while the climate policy mainly shifts employment from coal to clean energy production.

4.3 Resource stage

Figure 4 shows the equilibrium extraction paths of coal and oil/gas for both scenarios. Under laissez-faire, our model predicts an annual extraction of 7.3 Gt coal in the baseline



(a) Coal extraction

(b) Oil/Gas extraction

Figure 4: Global extraction of exhaustible resources: Optimal vs. laissez-faire solution.

period which is close to the empirical value of 7.2 Gt reported by the U.S. Energy Information Administration 2014 (EIA).[21] In the following periods, as shown by Figure 4(a) coal extraction grows significantly over the entire time interval, although the annual growth rate decreases from 1.7% in the initial periods to 1.2% at the end of the time window. Qualitatively, this result replicates the one by GHKT (cf. their Figure 3 on page 72). Such an unbounded extraction path is possible only under the assumption of a backstop technology which provides an equivalent substitute for coal some time in the future implying that there is no scarcity rent on this resource. As current estimates of global coal reserves reported by the EIA amount to approximately 890 Gt which would be exhausted in $t = 2075$, the required backstop technology would need to arrive within the next 60 years.[22] The role of this assumption is further explored in Section 5. Since initial coal extractions of 7.3 Gt predicted by the model match their empirical counterparts almost perfectly, we infer that the future adoption of a backstop technology might already be incorporated in empirically observed coal prices.[23]

Introducing the emissions tax reduces coal extraction instantaneously by 69% in the initial period with subsequent extractions remaining somewhat constant around 2.1 Gt p.a. on average over the next 200 years and gradually declining to zero thereafter. This almost flat extraction path is due to an increasing carbon tax over time which grows at

[21]U.S. Energy Information Administration. 2011, Annual Energy Review. Total primary coal production. Available from `http://www.eia.gov/totalenergy/data/annual/index.cfm`.

[22]U.S. Energy Information Administration. International Energy Statistics (2011), available from `http://www.eia.gov/beta/international/data/browser`.

[23]U.S. Energy Information Administration. International Energy Statistics (2014). Total primary coal production. Available from `http://www.eia.gov/beta/international/data/browser`.

the same rate as output. As a consequence, coal extraction along the efficient equilibrium does not exhaust the empirical stock of coal reserves reported above and the absence of a scarcity rent does not require the existence of a backstop technology.

As for the extraction of oil/gas shown in Figure 4(b), matters are different because this resource is always fully depleted. Under laissez-faire, the model predicts an annual extraction equal to 2.4 Gt p.a. over the next five decades which is reduced to 2.0 Gt p.a. under optimal taxation.[24] Moreover, in both equilibria extraction paths reach an interior maximum at some point in the future which is in line with empirical predictions[25], commonly referred to as 'peak-oil'.[26] In our model, the extraction-peak is reached in $t = 2055$ under laissez-afire and in $t = 2075$ under optimal taxation. Intuitively, the tax on emissions discourages oil usage in production which has to be counteracted by a lower oil price to ensure complete depletion of the resource. This is precisely the forgotten supply side argument advanced by Sinn (2012), see also Harstad (2012a). As a consequence, the tax merely pushes oil extraction back into the future by lowering it in the initial periods and increasing it after the peak with the total amount extracted unchanged.

4.4 Climate stage

We employ two measures to quantify climate change. First, total climate damage expressed as a percentage loss of potential world output which is directly given by (13).[27] Second, the increase in global mean temperature relative to the pre-industrial level. Preliminary data from the World Meteorological Organization (2016) shows that the global temperature in 2016 already exceeded the pre-industrial level by approximately 1.2 °C. Thus, the two-degree target set by the Paris accord on climate change in 2015 (United Nations Framework Convention on Climate Change (2015)) corresponds to an increase of at most 0.8 °C relative to the baseline period in our model. To compute this increase, we hypothesize a logarithmic relationship between atmospheric CO_2 concentration and global mean surface temperature, as proposed in Nordhaus & Yang (1996). Formally, we follow GHKT to determine global temperature in period t using the so-called Arrhenius

[24] According to empirical observations by the EIA (2014) (Available from `http://www.eia.gov/beta/international/data/browser`), annual production of oil and gas was roughly 6.4 Gt in 2014, a value significantly underpredicted by our model. However, the empirically measured quantity also includes oil and especially gas demand for heating by services, commerce, and residential. In our model, this direct use of resources by the private sector is ignored, which leads to a lower total demand of gas and lower extraction of the combined resource.

[25] See for instance Edwards, J. D. (2001).

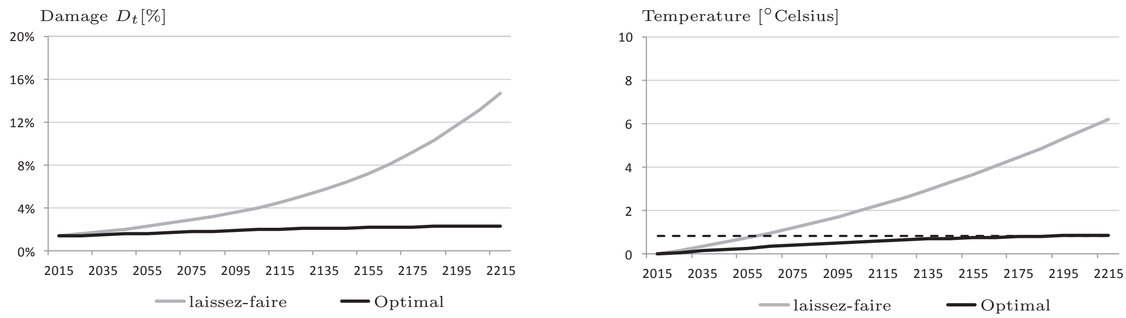
[26] This fact is not captured by the analysis in GHKT which predicts strictly decreasing extraction paths of oil implying that the global economy has already exceeded peak-oil.

[27] Using (1) to define $Y_t^{\ell,\text{pot}} := Y_t^\ell/(1 - D_t^\ell)$, one observes that D_t^ℓ can directly be interpreted as a percentage loss of potential output $Y_t^{\ell,\text{pot}}$ in region ℓ. Thus, under homogeneous climate damages, $D_t^\ell \equiv D_t$ is the percentage of potential world output $Y_t^{\text{pot}} := \sum_{\ell\in\mathbb{L}} Y_t^{\ell,\text{pot}}$ lost due to climate damages.

relation

$$TEMP_t = 3\log\left(\frac{S_t}{\bar{S}}\right)/\log 2. \tag{41}$$

Figure 5 depicts the evolution of the previously defined variables under both policy scenarios. Under optimal taxation, climate damages depicted in Figure 5(a) are contained



(a) Percentage losses of world GDP (b) Global temperature relative to 2015

Figure 5: Global damage and temperature: Optimal vs. laissez-faire solution.

to be less than 2.3% of world GDP throughout and even smaller during the initial periods. By contrast, they grow exponentially and become increasingly severe under laissez-faire, resulting in a loss of potential world GDP of almost 5% after 100 years in $t = 2115$ and up to 15% at the end of the time window in $t = 2215$. This quantifies our previous insights that unabated climate change leads to large damage and losses in productivity. A similar result is conveyed by Figure 5(b) which depicts the evolution of global temperature with the dashed line representing the aforementioned 2°C target. Under optimal taxation, global temperature increases by only 0.6 °C relative to 2015 until 2115. For longer time horizons, the increase continues to be small, reaching 0.86°C at the end of the simulation horizon in $t = 2215$. These numbers are again dramatically different under laissez faire. For this scenario, temperature increases by 2.3°C over the next 100 years (3.5°C relative to pre-industrial level) and by 6.2°C until $t = 2215$ (7.4°C relative to pre-industrial level). Quantitatively, these findings are in close conformity with the fifth assessment report by the Intergovernmental Panel on Climate Change (2015). This study asserts that the temperature increase relative to the pre-industrial level can still be limited to roughly 2 °C if strict climate policies are adopted while the 'global climate budget' will be exhausted within the next 30 years if no actions are taken. In fact, our model confirms that the two degree target will be exceeded after 40 years, in $t = 2065$, if emissions are not taxed.

4.5 A Pareto-improving transfer policy

All of the previous results involve only the aggregate equilibrium (23) and, therefore, are independent of the transfer policy $\theta = (\theta^\ell)_{\ell\in\mathbb{L}}$ which determines how tax revenue is distributed across regions. Any such transfer policy induces a unique equilibrium distribution $\mu = (\mu^\ell)_{\ell\in\mathbb{L}}$ of world consumption across regions where μ^ℓ is the constant share of world consumption in region ℓ.

In what follows, we focus on a particular transfer policy under which each region attains the same consumption share as under laissez-faire. This policy was shown in Hillebrand & Hillebrand (2017) to Pareto-improve the laissez-faire equilibrium if all regions implement the optimal tax. Formally, let $\mu^{\mathrm{LF}} = (\mu^{\ell,\mathrm{LF}})_{\ell\in\mathbb{L}}$ denote the consumption shares along the laissez faire equilibrium allocation ξ^{LF} determined by (16) and $T^{\mathrm{eff}} := \sum_{t=0}^{\infty} q_t \tau_t^{\mathrm{eff}} \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^\ell$ the total discounted tax revenue along the efficient aggregate allocation ξ^{eff}. Define the transfer share of region $\ell \in \mathbb{L}$ as

$$\theta^\ell := \frac{\mu^{\ell,\mathrm{LF}}\left[\sum_{k\in\mathbb{L}}\left(W^{k,\mathrm{eff}} + \Pi^{k,\mathrm{eff}} + r_0^{\mathrm{eff}} K_0^k\right) + T^{\mathrm{eff}}\right] - W^{\ell,\mathrm{eff}} - \Pi^{\ell,\mathrm{eff}} - r_0^{\mathrm{eff}} K_0^\ell}{T^{\mathrm{eff}}}. \qquad (42)$$

Transfers received by region ℓ are then determined as $T^\ell := \theta^\ell T^{\mathrm{eff}}$. Using (16), one verifies that the transfer policy (42) indeed yields the consumption share $\mu^{\ell,\mathrm{LF}}$ also at the efficient equilibrium under optimal taxation. Observe that θ^ℓ may be negative, in which case region ℓ imposes a lump sum tax on domestic consumers to finance transfer payments to the other region. Our numerical analysis presented in the next section shows that this case becomes relevant if climate damages are sufficiently heterogeneous.

4.6 Transfers under heterogeneous climate damages

Several studies suggest that climate damages are more severe in less developed countries, see World Bank (2010). Possible reasons are an increased vulnerability due to geographic differences and disadvantages due to a less developed capital stock and inferior knowledge for adaption (Bretschger & Suphaphiphat (2014)).

To incorporate this argument into our study, we add three additional cases to our previous benchmark parametrization leading to four different scenarios. Scenario A corresponds to the previous case with homogeneous climate damages while the remaining scenarios assume that climate damages in NOECD countries are slightly (Scenario B), significantly (Scenario C), and dramatically (Scenario D) higher than in OECD countries, respectively. Formally, this is achieved by varying the damage parameters γ^ℓ in (13). The following table displays these parameter variations together with the resulting consumption shares in the laissez faire equilibrium which are implemented by the transfer policy (42) along the efficient equilibrium.

In the benchmark Scenario A, consumption shares essentially coincide with the shares of world GDP produced in each region. As NOECD countries are increasingly exposed to

	Damage parameters			Consumption shares (LF)	
Scenario	$\gamma_1 \cdot 10^5$	$\gamma_2 \cdot 10^5$	Mean	OECD	NOECD
A: No differences	5.3	5.3	5.3	69.90%	30.20%
B: Small differences	4.4	6.2	5.3	70.37%	29.63%
C: Medium differences	3.6	7.0	5.3	70.74%	29.26%
D: Large differences	2.1	8.5	5.3	71.47%	28.53%

Table 2: Parameter variations and target consumption shares.

climate damages, however, their share of world consumption gradually reduces and is up to 1.6 percentage points smaller than in the benchmark case in the most extreme Scenario D. Thus, heterogeneous climate damages lead to a more uneven world consumption distribution under laissez faire on which our transfer scheme is based.

For each scenario, Table 3 displays the tax revenue in each region as a percentage of global revenue together with the transfer shares θ^1 and θ^2 defined by (42) required to induce the target consumption shares from Table 2. The net transfer displayed in the last column is the difference between tax revenue collected in and transfers received by OECD countries (both expressed as percentages of global tax revenue).

	Tax revenue		Transfer shares		Net Transfer
Scenario	OECD	NOECD	OECD	NOECD	OECD $\rightarrow$ NOECD
A	65.18%	34.82%	28.96%	71.04%	36.22%
B	65.01%	34.99%	76.30%	23.70%	-11.29%
C	64.67%	35.33%	109.55%	-9.55%	-44.88%
D	64.33%	35.67%	182.74%	-82.74%	-118.41%

Table 3: Tax revenue and optimal transfers between OECD and NOECD countries.

In the benchmark scenario with homogeneous damages, OECD countries collect slightly more than 65% of global tax revenue. To realize the target consumption shares shown in Table 2, they only need about 29% of these revenues resulting in a net transfer of more than 36% of global tax revenue to NOECD countries. In the additional three scenarios B-D, OECD countries continue to collect about 64% of total tax revenue which declines only slightly as climate damages are more biased towards NOECD countries. As is evident from Table 2, however, OECD countries are now entitled to receive a higher share of global consumption and, therefore, claim a higher share of tax revenue. Even in Scenario B where differences in climate damage are still moderate, this causes the direction of net transfers to reverse with OECD countries now claiming more than 76% of global tax revenue resulting in a net transfer of 11% from NOECD to OECD countries. This effect continues and becomes even more extreme in Scenarios C and D as climate damages become more severe in NOECD countries. In these cases, the transfers needed to realize the target consumption distribution exceed the tax revenue of NOECD countries which must now impose a lump-sum tax (corresponding to a negative transfer) on their domestic consumers to finance these transfer payments.

The intuition for this somewhat disturbing result is that climate effects in OECD countries are small if not negligible in these scenarios while NOECD countries suffer dramatically. Thus, the latter countries benefit relatively more from the climate tax and must share this benefit with OECD countries via transfers to retain the world consumption distribution. Loosely speaking, NOECD countries must 'pay to survive' under the proposed transfer policy. The most extreme version of this scenario would be the case where only poor countries are affected by climate damages and rich countries must be incentivized to take action against climate change via transfers. These results are of course a direct consequence of our transfer scheme which is based on the laissez faire distribution of consumption. Also recall that all regions benefit from such a climate policy and attain utility strictly higher than under laissez faire. The general insight is that countries severely affected by climate damages are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfer payments.

The previous results motivate a modification of our transfer scheme to incorporate heterogeneities of climate damages of different regions. As such a scheme may potentially make some regions worse off, the proposed modification requires a more elaborate analysis of the incentives of regions to implement such a transfer policy, as is done, e.g., in Eyckmans & Tulkens (2003). Such an extended analysis is reserved for future research.

5 An Alternative Scenario of Climate Change

5.1 Backstop technology

A crucial assumption in the previous simulation study is the absence of a scarcity rent on coal. Formally, this is accomplished by assuming the existence of a backstop technology which offers a perfect substitute for coal in the future rendering coal resources essentially infinite. The first one to introduce this idea was Nordhaus (1973), who used it as a simple justification for the absence of any resource constraints on fossil fuels in the DICE/RICE framework. Although widely used and analyzed in subsequent studies, (cf. Tahvonen & Salo (2001), Tsur & Zemel (2005), Chakravorty, Leach & Moreaux (2012), Valente (2011), Golosov, Hassler, Krusell & Tsyvinski (2014)), the assumption remains somewhat controversial, for at least two reasons. First, such a technology still has to be developed some time in the future and the exhaustibility of coal vanishes *only if* such a technology actually arrives. Second, the technology to be developed has to offer a perfect substitute for coal (or other fossil fuels) with respect to both its role in energy production as well as its carbon content. This seems a fairly restrictive assumption.

For this reason, the present section analyzes the quantitative effects of a backstop technology and how dropping this assumption affects the previous results. Formal, we now assume that coal also has a finite resource stock, i.e., $R_{2,0} < \infty$ chosen to match empirical data. According to estimates reported by the U.S. Energy Information Administration (EIA) in 2014, global resources of coal (anthracite and lignite) amount to 890 Gt of which 46% are located in OECD countries. We thus set $R^1_{0,2} = 414$ and $R^2_{0,2} = 476$ Gt.

The initial coal price $v_{2,0}$ is now chosen such that equilibrium coal extractions are compatible with the global resource stock. For the optimal equilibrium, this implies no change and the same choice $v_{2,0} = c_2$ as before, because the stock of coal set above is not exploited at equilibrium. In the laissez faire case, however, the initial resource prices need to be adjusted to 356 \$/t (49.8 \$/bbl) for oil/gas and 48.6 \$/t for coal which are close to empirical observations reported by the World Bank (2015a). The other parameter values are kept at the same values as in table 1.[28]

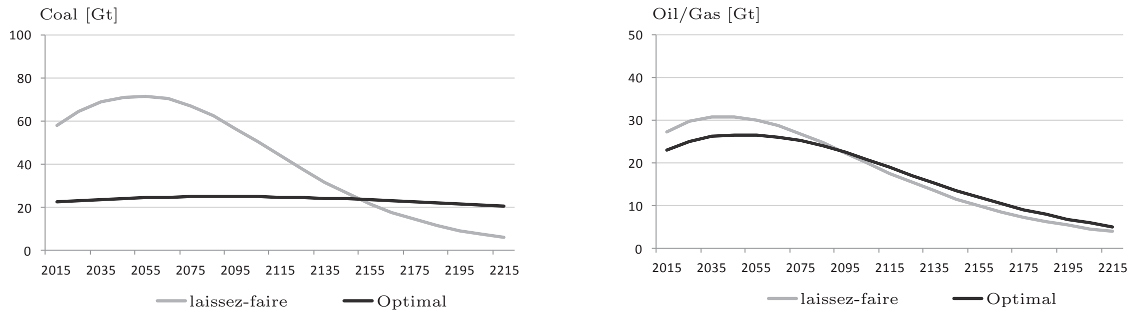
The following sections compare the 'no backstop' laissez faire equilibrium and the (unchanged) optimal equilibrium for the resource and the climate stage.

5.2 Resource stage

Figures 6(a) and 6(b) depict the predicted extraction paths of coal and oil/gas in the absence of a backstop technology. For the baseline period, the annual extraction of coal is 5.8 Gt under laissez faire and reduces to 2.2 Gt under the optimal climate policy. Thus, the climate tax reduces fossil fuel consumption in the initial periods and postpones extraction farther into the future. More importantly, however, the absence of a backstop leads to a sizeable reduction of coal consumption relative to the backstop case studied in Section 4.3 (cf. Figure 4) even under laissez faire. For this case, accumulated coal extractions amount to 404 Gt (about 45% of total resources) over the next 50 years and to 868 Gt (97%) over the next 200 years. Quantitatively, however, these extractions seem too low compared to empirical observations (the U.S. Energy Information Administration (2011) reports annual extractions of 7.3 Gt).[29] This seems to support the view that markets do expect a backstop technology to become available in the future and coal reserves are depleted accordingly.

For oil and gas, the model predicts annual extractions equal to 2.7 Gt over the next decade under laissez-faire which are slightly reduced 2.3 Gt under optimal taxation.

[28] Since profits from the coal sector are positive at the laissez faire equilibrium, the world distribution of coal reserves is relevant for calculating the regional shares of global consumption under laissez faire, on which optimal transfers are based. Compared to the previous results rom Section 4.6, however, the world consumption distribution changes only marginally. This holds even if coal reserves were completely located in one of the two regions. Thus, our results are robust against arbitrary regional



(a) Coal extraction

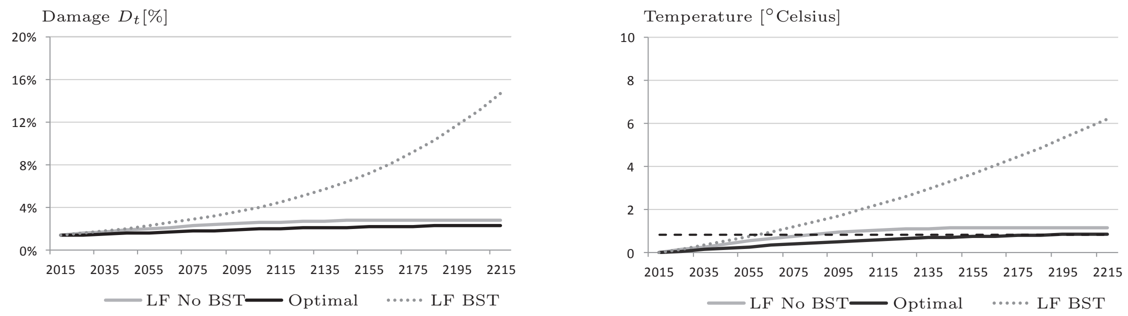
(b) Oil/Gas extraction

Figure 6: Global extraction of exhaustible resources: Optimal vs. laissez-faire solution.

Thus, the tax again lowers initial extractions which are postponed into the future and exceed laissez faire extractions after $t = 2115$. Each equilibrium path exhibits again a 'peak-oil' point at which extractions become maximal and which the carbon tax shifts farther into the future.

5.3 Climate stage

The climate effects under both policies are portrayed by Figures 7(a) and 7(b) which show the evolution of total damages and global temperature defined as in Section 4.4. The most striking difference relative to Figure 5 is that climate damages are now con-



(a) Percentage losses of world GDP

(b) Global temperature relative to 2015

Figure 7: Global temperature and climate damages: Optimal vs. laissez-faire solution.

tained and remain below 3% over the next 200 years even under laissez faire. While a carbon tax reduces these damages further to a maximum of 2.3%, the gain is much

distributions of global coal reserves. For this reason, optimal transfers are almost exactly as before.

[29]U.S. Energy Information Administration. 2011, Annual Energy Review. Total primary coal production. Available from `http://www.eia.gov/totalenergy/data/annual/index.cfm`.

smaller in the absence of backstop technology. The same result is conveyed by the evolution of temperature shown in Figure 7(b) with the the dashed line representing again the two-degree target. While temperature increases significantly under laissez faire and exceeds the two-degree target in $t = 2095$, this increase is limited to a maximum of three degrees relative to pre-industrial level and therefore much smaller than in the presence of a backstop technology. Thus, the effects of climate change are still sizeable and justify political intervention but the cost of no intervention is much smaller compared to the scenario from Section 4.4. This also shows that the extreme results from GHKT (cf. their Figures 6 and 7) are mainly driven by the assumption of a backstop technology and become much less extreme if such a technology is not assumed.

While these results confirm again that burning fossil fuel, particularly coal, poses a serious threat to the global economy and call for immediate political action, they also show that the quantitative results from Section 4 and also in GHKT tend to overstate these damages, as they crucially hinge on the (highly speculative) emergence of a backstop for coal some time in the future. This leads to the somewhat paradoxical result that a more pessimistic view of technological possibilities in the future (no backstop technology becomes available) implies a more optimistic prospect of future climate damages.

6 Conclusions

The multi-region model employed in this paper permits to study the impact of alternative policies on climate variables and consumption, production, and resource extraction in each region. The numerical algorithm developed permits to compute equilibrium allocations in this framework under arbitrary climate emissions tax policies and in the presence of arbitrarily many regions and production sectors. Our numerical study quantifies the effects of alternative climate policies for OECD and NOECD countries. Climate policies under scrutiny represent the two extreme scenarios where either no action is taken against climate change (laissez-faire) or the optimal (Pigovian) tax policy derived in Hillebrand & Hillebrand (2017) is implemented.

Our quantitative results confirm the general intuition that measures against climate change come at some initial costs but pay-off greatly in the future while not implementing such measures may have disastrous consequences. Under optimal taxation, short-run losses are shown to be between 0.5 and 1.2 percent of domestic GDP in both regions with relative losses for NOECD countries about twice as large as in the OECD region. Temperature increases remain moderate and broadly in line with the two-degree target over the next 100 years. Under laissez faire, the results depend crucially on whether or not a backstop technology is assumed. If such a technology emerges within the next 100 years, coal extraction grows exponentially and leads to devastating outcomes. While still significant, these damages are much smaller if no such backstop technology is assumed, stressing the crucial role of this assumption as the key driver for the quantitative findings in GHKT and other numerical studies of climate change.

Optimal transfer payments are chosen to preserve the world income distribution under laissez faire which induces a Pareto-improvement that makes each region better off. This result is independent of whether or not a backstop technology is assumed and may be interpreted as a participation constraint for each region to implement the optimal tax. We view this as a minimal requirement to address the free-riding problem associated with climate negotiations. Germaint et al. (2003) refer to this constraint as 'individual rationality'. Quantitatively, our study finds that homogeneous climate damages imply moderate transfer payments from OECD to Non-OECD countries while these payments are reversed if climate damages are biased towards Non-OECD countries. The intuition for this somewhat disturbing result is that countries facing larger damages from climate change are in desperate need of climate policies implemented by other regions and, therefore, have little bargaining power in the political process determining transfers.

While still inducing a Pareto-improvement in these regions, this result stresses the need to adopt a more elaborate study of the bargaining process as a cooperative or non-cooperative game between regions as, e.g., in Harstad (2012b) or Harstad (2016). Such an extension constitutes a first and major objective of future research. A second, more empirical goal is to include more regions and a finer specification of energy sectors in the analysis as in Nordhaus & Yang (1996) permitting to quantify sectoral changes and transfers across regions at a more disaggregated level. Both the framework and the computational algorithm developed are directly amendable to such extensions. Additional extensions which require modifications of the current framework are to include abatement costs and uncertainty, e.g., in climate damages and parameters, as well as endogenizing technological progress in the model as in Acemoglu et al. (2012) or van den Bijgaart (2017) who both study the case with directed technical change.

A Computational Details

In this section we explain the details of our algorithm to solve the M-dimensional problem $\Phi(\xi^1, \lambda) = 0$ defined in Section 2.2 for some vector $\xi^1 \in \Xi^1$ given a fixed vector $\lambda = (\mathbf{N}^s, \mathbf{Q}, \mathbf{v}_{-1}, \mathbf{S}_{-1}, \bar{C}_{-1}, \bar{K}) \in \Lambda$ of pre-determined variables. For convenience, the time index t is suppressed in this section.

A.1 The general idea

Our algorithm is based on a very simple idea which is illustrated here for $M = 3$. Suppose we want to determine three real numbers $(x_0, y_0, z_0) \in \mathbb{X} \times \mathbb{Y} \times \mathbb{Z}$ such that $\Phi_1(x_0, y_0, z_0) = \Phi_2(x_0, y_0, z_0) = \Phi_3(x_0, y_0, z_0) = 0$. Then, we can break up this problem into three nested subproblems, referred to as stages, where each stage uses the results from the previous ones.

Stage I: Given arbitrary numbers $\hat{y} \in \mathbb{Y}$ and $\hat{z} \in \mathbb{Z}$, consider the problem of determining $\hat{x} \in \mathbb{X}$ such that $\Phi_1(\hat{x}, \hat{y}, \hat{z}) = 0$. If this problem admits a unique solution for any $\hat{y} \in \mathbb{Y}$ and $\hat{z} \in \mathbb{Z}$, we can define a function $\phi_x : \mathbb{Y} \times \mathbb{Z} \longrightarrow \mathbb{X}$ which determines $\hat{x} = \phi_x(\hat{y}, \hat{z})$ such that $\Phi_1(\phi_x(\hat{y}, \hat{z}), \hat{y}, \hat{z}) = 0$ for any $(\hat{y}, \hat{z}) \in \mathbb{Y} \times \mathbb{Z}$.

Stage II: Given some value $\tilde{z} \in \mathbb{Z}$, consider the problem of choosing $\tilde{y} \in \mathbb{Y}$ such that $\Phi_2(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ where $\tilde{x} = \phi_x(\tilde{y}, \tilde{z})$ is determined by the previous stage. In other words, given $\tilde{z} \in \mathbb{Z}$ we determine a unique $\tilde{y} \in \mathbb{Y}$ such that $\Phi_2(\phi_x(\tilde{y}, \tilde{z}), \tilde{y}, \tilde{z}) = 0$. If this is again possible for any $\tilde{z} \in \mathbb{Z}$, we can define the solution as a function $\phi_y : \mathbb{Z} \longrightarrow \mathbb{Y}$ such that $\tilde{y} = \phi_y(\tilde{z})$.

Stage III: Consider the problem of choosing $\check{z} \in \mathbb{Z}$ such that $\Phi_3(\check{x}, \check{y}, \check{z}) = 0$ where $\check{y} = \phi_y(\check{z})$ and $\check{x} = \phi_x(\check{y}, \check{z})$ are again determined by the functions derived on the previous two stages. If such a solution exists and is unique, setting $z_0 = \check{z}$, $y_0 = \phi_y(z_0)$, and $x_0 = \phi_x(y_0, z_0)$ is the unique solution to the original problem.

A.2 The algorithm

Our solution approach corresponds exactly to the three-stage structure motivated in the previous example except that the solution sets $\mathbb{X}$, $\mathbb{Y}$, $\mathbb{Z}$ and the functions Φ_1, Φ_2, and Φ_3 are higher-dimensional.

Stage I: Given an arbitrary capital allocation $\hat{\mathbf{K}} = (\hat{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{K} := \mathbb{R}_{++}^{L(I+1)}$ and some resource allocation $\hat{\mathbf{X}} = (\hat{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X} := \mathbb{R}_{++}^{LI_x}$, consider the problem of determining a labor allocation[30] $\hat{\mathbf{N}} = (\hat{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{M} := \mathbb{R}_{++}^{L(I+1)}$ which solves the conditions defined by equations (4b), (6b), (29b), and (19) which (after eliminating the wage w^ℓ and energy prices p_i^ℓ) can be stated for all $\ell \in \mathbb{L}$ as

$$\begin{aligned}
\partial_N F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) &= Q_i^\ell \partial_N F_i(\hat{N}_i^\ell, \hat{K}_i^\ell, \hat{X}_i^\ell) \partial_{E_i} F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) \; \forall i \in \mathbb{I}_x &\text{(A.1a)}\\
&= Q_i^\ell \partial_N F_i(\hat{N}_i^\ell, \hat{K}_i^\ell) \partial_{E_i} F_0(\hat{N}_0^\ell, \hat{K}_0^\ell, (\hat{E}_i^\ell)_{i\in\mathbb{I}}) \; \forall i \in \mathbb{I} \backslash \mathbb{I}_x &\text{(A.1b)}\\
\sum_{i\in\mathbb{I}} N_i^\ell &= \bar{N}^\ell. &\text{(A.1c)}
\end{aligned}$$

Energy inputs $(\hat{E}_i^\ell)_{i\in\mathbb{I}}$ in (A.1) are determined from $\hat{\mathbf{N}}$, $\hat{\mathbf{K}}$, and $\hat{\mathbf{X}}$ by (3) and (5) for all $\ell \in \mathbb{L}$. Note that climate damage $1 - D^\ell(\hat{\mathbf{X}}, \mathbf{S}_{-1})$ and final sector productivity Q_0^ℓ enter as multiplicative terms in all conditions in (A.1a) and (A.1b) and, therefore, cancel out.

The system (A.1) involves $I + 1$ equations for each region $\ell \in \mathbb{L}$. Define the function $\Phi_1 : \mathbb{M} \times \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{R}^{L(I+1)}$ such that, given $\hat{\mathbf{K}}$ and $\hat{\mathbf{X}}$, $\hat{\mathbf{N}}$ solves (A.1) if and only if

[30] We define the set of feasible labor allocations as $\mathbb{M}$ because $\mathbb{N}$ is usually reserved for the natural numbers.

$\Phi_1(\hat{\mathbf{N}}, \hat{\mathbf{K}}, \hat{\mathbf{X}}) = \mathbf{0}$. If such a solution exists and is unique for any $(\hat{\mathbf{K}}, \hat{\mathbf{X}}) \in \mathbb{K} \times \mathbb{X}$, we can define a function $\phi_N : \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{M}$ which determines the solution $\hat{\mathbf{N}} = \phi_N(\hat{\mathbf{K}}, \hat{\mathbf{X}})$.

To actually compute $\hat{\mathbf{N}}$ in our simulations, we exploit that (A.1) can be solved separately for each region and adopt the following algorithm to compute the solution $\hat{\mathbf{N}}^\ell$ for region $\ell \in \mathbb{L}$. Given a current candidate solution $\tilde{\mathbf{N}}^\ell$ satisfying (A.1c), we compute the associated marginal products of labor $\tilde{w}_i^\ell$ defined by the terms in (A.1a) and (A.1b) in each sector $i \in \mathbb{I}_0$. We then determine the sector $i_{\max}$ with the highest and $i_{\min}$ with the lowest 'wage' and record the current amount of labor allocated to sector $i_{\max}$ as a lower bound and the amount allocated to sector $i_{\min}$ as an upper bound for the actual solution $\hat{N}_i^\ell$ in these sectors. Further, we adjust $\tilde{\mathbf{N}}^\ell$ by shifting a certain amount of labor (which depends on the bounds computed) from the lowest to the highest paying sector. This produces a new candidate solution, for which the process is repeated. Convergence is obtained if all sectors pay the same wage. In our simulations, this approach proved to be a reliable way of solving (A.1).

Stage II: Given an arbitrary resource allocation $\tilde{\mathbf{X}} = (\tilde{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X}$, consider the problem of determining a capital allocation $\tilde{\mathbf{K}} = (\tilde{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} \in \mathbb{K}$ which solves the conditions defined by equations (4a), (6a), (29), and (20) which (after eliminating the capital return r and energy prices p_i^ℓ) can be stated as

$$(1 - D^\ell(\tilde{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^\ell \partial_K F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) = (1 - D^k(\tilde{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^k \partial_K F_0(\tilde{K}_0^k, \tilde{N}_0^k, (\tilde{E}_i^k)_{i\in\mathbb{I}}) \tag{A.2}$$

for all $\ell, k \in \mathbb{L}$, $\ell \neq k$ and

$$\begin{aligned}
\partial_K F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) &= Q_i^\ell \partial_K F_i(\tilde{K}_i^\ell, \tilde{N}_i^\ell, \tilde{X}_i^\ell) \partial_{E_i} F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) \ \forall i \in \mathbb{I}_x && \text{(A.3a)}\\
&= Q_i^\ell \partial_K F_i(\tilde{K}_i^\ell, \tilde{N}_i^\ell) \partial_{E_i} F_0(\tilde{K}_0^\ell, \tilde{N}_0^\ell, (\tilde{E}_i^\ell)_{i\in\mathbb{I}}) \ \forall i \in \mathbb{I}\backslash\mathbb{I}_x && \text{(A.3b)}\\
\sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_0} K_i^\ell &= \bar{K} && \text{(A.3c)}
\end{aligned}$$

for all $\ell \in \mathbb{L}$. Here, $\tilde{\mathbf{N}} = (\tilde{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_N(\tilde{\mathbf{K}}, \tilde{\mathbf{X}})$ is determined by Stage I and energy inputs $(\tilde{E}_i^\ell)_{i\in\mathbb{I}}$ in both (A.2) and (A.3) follow from (3) and (5) for all $\ell \in \mathbb{L}$. Equation (A.2) equates marginal products of capital in final production across all regions while (A.3) equates marginal products of capital across all sectors within each region. Both conditions together thus equalize capital returns across all sectors and regions.

The system defined by (A.2) and (A.3) consist of $L - 1 + LI + 1 = L(I + 1)$ equations. Define the function $\Phi_2 : \mathbb{K} \times \mathbb{X} \longrightarrow \mathbb{R}^{L(I+1)}$ such that, given $\tilde{\mathbf{X}}$, $\tilde{\mathbf{K}}$ solves (A.2) and (A.3) if and only if $\Phi_2(\tilde{\mathbf{N}}, \tilde{\mathbf{K}}, \tilde{\mathbf{X}}) = \mathbf{0}$ where $\tilde{\mathbf{N}} = \phi_N(\tilde{\mathbf{K}}, \tilde{\mathbf{X}})$. If such a solution exists and is unique for any $\tilde{\mathbf{X}} \in \mathbb{X}$, there exists a function $\phi_K : \mathbb{X} \longrightarrow \mathbb{K}$ which determines this solution as $\tilde{\mathbf{K}} = \phi_K(\tilde{\mathbf{X}})$, i.e., $\Phi_2(\phi_N(\phi_K(\tilde{\mathbf{X}}), \tilde{\mathbf{X}}), \phi_K(\tilde{\mathbf{X}}), \tilde{\mathbf{X}}) = \mathbf{0}$ for all $\tilde{\mathbf{X}} \in \mathbb{X}$.

To compute $\tilde{\mathbf{K}}$, it is tempting to follow a similar strategy as for Stage I, defining for any candidate solution $\check{\mathbf{K}}$ the marginal capital products $\check{r}_i^\ell$ in each region and sector and shifting capital from the lowest to the highest paying sector. Due to (A.3c), this problem can no longer be solved separately for each region but requires shifting capital around globally across all regions and sectors. Unfortunately, this approach caused some potential instability for our algorithm which, occasionally, did not converge to the desired solution. To remedy this problem, we therefore break up Stage II into two steps.

In the first step, we fix a capital distribution $(\bar{K}^\ell)_{\ell\in\mathbb{L}} \in \mathbb{R}^L_{++}$ across regions where $\bar{K}^\ell > 0$ is total capital employed in region ℓ and $\sum_{\ell\in\mathbb{L}} \bar{K}^\ell = \bar{K}$. We then solve (A.3) separately for each region ℓ, replacing (A.3c) by the condition

$$\sum_{i\in\mathbb{I}_0} K_i^\ell = \bar{K}^\ell. \tag{A.4}$$

This step allows us to essentially employ the same routine as on Stage I and determine for each region $\ell \in \mathbb{L}$ a capital allocation $\check{\mathbf{K}}^\ell = (\check{K}_i^\ell)_{i\in\mathbb{I}_0}$ which induces a uniform capital return $\check{r}^\ell$ across sectors.

In the second step, we adjust the capital distribution $(\bar{K}^\ell)_{\ell\in\mathbb{L}}$ based on the regional capital returns $\check{r}^\ell$ computed in Step 1 shifting capital from the lowest to the highest paying region and then repeating the computation of Step 1. The desired solution is reached if equation (A.2) is satisfied and capital returns are identical for all regions.

It turned out that this two-step strategy completely eliminates the previous convergence problems.

Stage III: At the final stage, we determine the resource allocation $\check{\mathbf{X}} = (\check{X}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \in \mathbb{X}$ such that equations (4c), (7), and (26) are satisfied. After eliminating energy prices using (2c), we obtain the following condition which must hold for all $\ell \in \mathbb{L}$ and $i \in \mathbb{I}_x$:

$$(1 - D^\ell(\check{\mathbf{X}}, \mathbf{S}_{-1}))Q_0^\ell \partial_{E_i} F_0(\check{K}_0^\ell, \check{N}_0^\ell, (\check{E}_i^\ell)_{i\in\mathbb{I}}) Q_i^\ell \partial_X F_i\big(\check{K}_i^\ell, \check{N}_i^\ell, \check{X}_i^\ell\big) = c_i + \check{r}(v_{i,-1} - c_i) + \zeta_i \check{\tau} \tag{A.5}$$

where the factor allocation $\check{\mathbf{K}} = (\check{K}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_K(\check{\mathbf{X}})$ and $\check{\mathbf{N}} = (\check{N}_i^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} = \phi_N(\check{\mathbf{K}}, \check{\mathbf{X}})$ is determined from $\check{\mathbf{X}}$ by Stages I and II. The factor allocation determines energy inputs $(\check{E}_i^\ell)_{i\in\mathbb{I}}$ in (A.5) by (3) and (5), the tax rate $\check{\tau}$ by using (28) in (26), and the global capital return $\check{r}$ as the marginal product of capital in any region or sector.

Noting that the r.h.s. in (A.5) is independent of ℓ, the system (A.5) involves LI_x equations that can potentially be solved to determine a unique solution $\check{\mathbf{X}}$. Our solution strategy is to determine for any candidate solution $\acute{\mathbf{X}}$ the induced factor allocation $\acute{\mathbf{K}}$ and $\acute{\mathbf{N}}$ and the r.h.s. in (A.5) as $\acute{\pi}_i := c_i + \acute{r}(v_{i,-1} - c_i) + \zeta_i \acute{\tau}$ for each resource $i \in \mathbb{I}_x$. Then, for each $\ell \in \mathbb{L}$, we adjust X_i^ℓ based on the discrepancy between $\acute{\pi}_i$ and the marginal product on the l.h.s. in (A.5). This produces a new candidate solution for which the previous computations can be repeated until convergence obtains and the solution $\check{\mathbf{X}}$ is reached. ■

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